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A STUDY OF SUPERSONIC FLOW PAST VIBRATING SHELLS AND CASCADES

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THESIS

A STUDY OF SUPERSONIC FLOW
PAST VIBRATING SHELLS AND CASCADES

by

Christian William Brix, Jr.

Thesis Advisor: M. F. Platzer

June 1973

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A Study of Supersonic Flow
Past Vibrating Shells and Cascades

by

Christian William Brix, Jr.
Lieutenant, United States Navy
M.S., Naval Postgraduate School, 1972

Submitted in partial fulfillment of the
requirements for the degree of

AERONAUTICAL ENGINEER

from the

NAVAL POSTGRADUATE SCHOOL
June 1973

ABSTRACT

Supersonic flow past oscillating cylindrical shells and oscillating flat-plate cascades is analyzed using linearized characteristics methods. Pressure distributions and generalized aerodynamic forces are computed for arbitrary radius to length ratios, axial and circumferential wave numbers, Mach numbers, and reduced frequencies. Excellent agreement is obtained with a previous solution of the complete unsteady linearized potential equation by Dowell and Widnall using Laplace transform techniques. Also, pressure distributions are computed for flat-plate cascades with subsonic leading edge locus for arbitrary solidity, stagger angle, frequency, and interblade phase angle. For comparison, a two-blade solution is developed using the method of singularities. Good agreement is obtained for this special case.

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LIST OF SYMBOLS

A_1	See Equation (G-3)
C	Teipel complex velocity of sound perturbation amplitude.
c	Velocity of sound perturbation
$\bar{c}$	Local velocity of sound
c_∞	Free-stream velocity of sound
$\hat{c}$	Blade chord
C_p	Coefficient of pressure
$\bar{C}_p$	Pressure coefficient amplitude
d	Interblade distance
e	2.718282
$H_n^{(2)}$	Hankel function of the second kind, order n
h	Radial deflection (shell) Blade chord line (cascade)
i	The square root of minus one
J_n	Bessel function of the first kind, order n
k	Reduced frequency
L	Cylinder length
M	Free-stream Mach number
m	Axial mode number
N	Number of blades
n	Circumferential mode number
P	Non-dimensional complex pressure perturbation amplitude
p	Pressure
p_∞	Free-stream pressure

R	Radius to length ratio
r	Radial coordinate
S	Entropy
Δs	Increment of distance along a characteristic
T	Temperature
t	Time
U	Teipel complex streamwise velocity perturbation amplitude
u	Streamwise velocity perturbation
$\bar{u}$	Local streamwise velocity
u_∞	Free-stream velocity
V	Teipel complex normal velocity perturbation amplitude
v	Normal velocity perturbation
W	Velocity in the z-direction
w	z-direction velocity perturbation
x,y,z	Rectangular coordinates
x,r,Θ	Cylindrical coordinates
x_0	Elastic axis position
Y_n	Bessel function of the second kind, order n
$Z(x)$	Radial deflection amplitude
α	Mach angle
β	Stagger angle
δ	Interblade phase angle
ξ,η	Characteristic coordinates
ν	Grid fineness ratio
ρ	Local density
ρ_∞	Free-stream density

Φ	Velocity perturbation amplitude
ϕ	Velocity potential amplitude
$\psi_m(x)$	Stream-wise mode shape
ψ_1	See Equation (G-3)
ω	Circular frequency
π	3.141593

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I. INTRODUCTION

Supersonic flutter of skin panels first occurred on the German V-2 missile and has become of increasing engineering importance in recent years with the development of supersonic flight vehicles. Panel flutter, for example, was found to occur on the X-15 Rocket Aircraft and Saturn Space Launch Vehicle. An integral part of the flutter analysis of such panels is the determination of the oscillatory pressure distribution. A number of approaches have been formulated which resulted mostly in approximation theories. A detailed review of the state of the art can be found in Reference 6. Only one "exact" approach was worked out by Dowell and Widnall using the complete linearized unsteady potential equation (Ref. 2). However, this approach requires rather complicated and time-consuming complex integration techniques.

In the present work a method of characteristics procedure is developed to determine supersonic flow past vibrating panels and cylindrical shells resulting in a rapid computation of the pressure distribution and generalized aerodynamic forces.

A second flutter problem has also become of considerable interest very recently. Due to the continuing trend toward higher tip speeds in modern jet engine fans and compressors, the possibility of supersonic blade flutter has become of considerable concern. A recent review of this problem area

has been given in "A Workshop on Aeroelasticity in Turbomachines" (Ref. 13).

In the present study the method of characteristics is applied to compute supersonic flow past a finite flat plate cascade with subsonic leading edge locus. In the past, cascade studies have proved to be a useful tool for the understanding of the complicated three-dimensional, incompressible or subsonic flow through an actual compressor stage. Although it is doubtful that the much more complex mixed transonic-supersonic flow can be modeled by a two-dimensional cascade idealization, the analysis of an infinitely thin flat plate cascade may point the way toward a fuller understanding. The method of characteristics again proved to be a convenient tool for the analysis of supersonic flow past oscillating cascades.

II. PROBLEM FORMULATION

Inviscid adiabatic flow of a gas along the positive x-axis is considered which is perturbed by either A) an airfoil cascade executing small pitch oscillations (Fig. 1) or B) small amplitude vibrations of the surface of a circular cylinder whose axis is aligned with the free-stream direction (Fig. 2).

The following basic equations hold:

The equation of state for a perfect gas,

$$p = \rho R T \tag{1}$$

the conservation equation for mass,

$$\frac{D\rho}{Dt} + \rho\left(\frac{\partial \bar{u}}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0 \tag{2}$$

the conservation of momentum

$$\frac{D\bar{u}}{Dt} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0$$

$$\frac{Dv}{Dt} + \frac{1}{\rho}\frac{\partial p}{\partial y} = 0$$

$$\frac{Dw}{Dt} + \frac{1}{\rho}\frac{\partial p}{\partial z} = 0$$

and the conservation of energy

$$\frac{Ds}{Dt} = 0$$

where $\frac{D}{Dt}$ is the substantial derivative and s is the entropy of the fluid particle.

The assumption of small amplitude vibrations makes it possible to introduce the small perturbation concept.

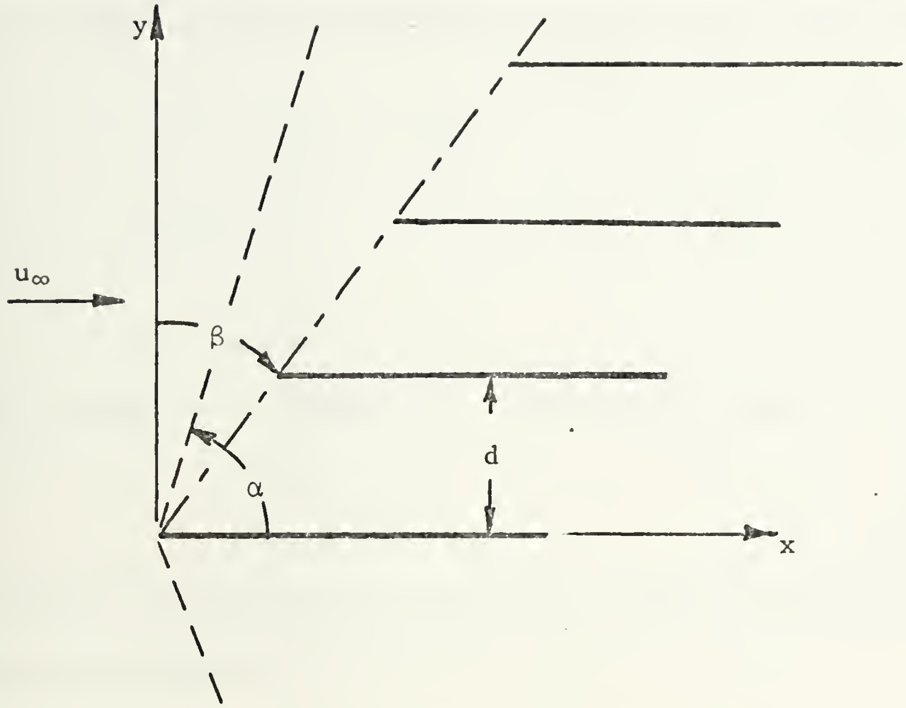


Figure 1. Cascade Configuration

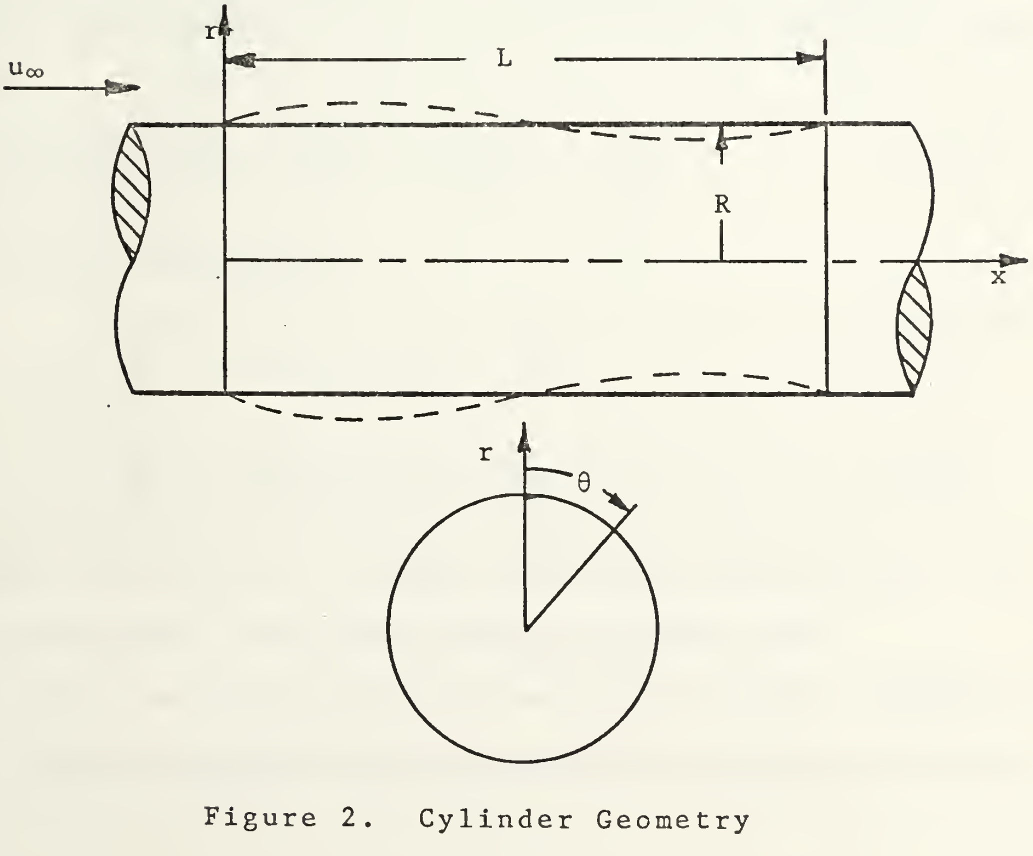


Figure 2. Cylinder Geometry

Therefore, all flow quantities are assumed as small perturbations linearly superimposed on free-stream quantities, i.e.,

$$\bar{u} = u_\infty + u \tag{5}$$

$$V = v \tag{6}$$

$$W = w \tag{7}$$

$$\bar{c} = c_\infty + c \tag{8}$$

and the pressure and density perturbations are:

$$\Delta p = p - p_\infty \tag{9}$$

$$\Delta \rho = \rho - \rho_\infty \tag{10}$$

The pressure perturbation can be expressed by the velocity of sound perturbation, i.e.,

$$p - p_\infty = \frac{2}{\gamma-1} \rho_\infty c_\infty (\bar{c} - c_\infty) \tag{11}$$

Specializing to planar flow, one has for the continuity equation:

$$\frac{2}{\gamma-1}\frac{\partial c}{\partial t} + \frac{2}{\gamma-1} u_\infty \frac{\partial c}{\partial x} + c_\infty \frac{\partial u}{\partial x} + c_\infty \frac{\partial v}{\partial y} = 0 \tag{12}$$

and the Euler equations:

$$\frac{\partial u}{\partial t} + u_\infty \frac{\partial u}{\partial x} + \frac{2}{\gamma-1} c_\infty \frac{\partial c}{\partial x} = 0 \tag{13a}$$

$$\frac{\partial v}{\partial t} + u_\infty \frac{\partial v}{\partial x} + \frac{2}{\gamma-1} c_\infty \frac{\partial c}{\partial y} = 0 \tag{13b}$$

Equations (12) and (13) are the governing equations for the case of supersonic flow past oscillating cascades.

For the case of supersonic flow past vibrating cylindrical shells, the equivalent single equation for the perturbation

potential, $\Phi(x,r,\theta,t)$ in cylindrical coordinates was taken as the more convenient basic governing equation, i.e.,

$$(1-M^2)\Phi_{xx}+\Phi_{rr}+\frac{1}{r}\Phi_r+\frac{1}{r^2}\Phi_{\theta\theta}-2\frac{M}{c_\infty}\Phi_{xt}-\frac{1}{c_\infty^2}\Phi_{tt} = 0 \qquad (14)$$

For a derivation of this equation reference is made to one of the standard texts, e.g., Bisplinghof, Ashley, and Halfman, "Aeroelasticity", (Ref. 1).

For a complete formulation, proper initial and boundary conditions must be prescribed. The most important ones are A) the flow on the body surface must be tangent at any instant in time and B) Sommerfeld's radiation condition must be satisfied, i.e., waves must propagate away from sources of disturbance.

After properly linearizing the general flow tangency condition, (see Ref. 1) one obtains for the flow tangency condition on a thin airfoil, performing a time dependent small amplitude motion about the y=0 plane,

$$v = \frac{\partial h}{\partial t}+u_\infty\frac{\partial h}{\partial x} \qquad \text{at } y=0 \qquad (15)$$

where h(x,t) is the equation of the airfoil chord line.

For the vibrating cylindrical shell the flow tangency condition over the flexible portion becomes

$$\Phi_r\Big|_{r=R} = \frac{\partial h}{\partial t}+u_\infty\frac{\partial h}{\partial x} \qquad 0\leq x\leq l$$

$$\Phi_r\Big|_{r=R} = 0 \qquad x<0 \qquad (16)$$

where h(x,θ,t) in this case describes the radial shell deflection.

With the assumption of a perturbation potential, the momentum equations, equations (3), may be integrated to

$$\frac{\partial \Phi}{\partial t}+\frac{u_\infty{}^2}{2}+u_\infty\frac{\partial \Phi}{\partial x}+\int\frac{dp}{\rho}=c(t) \tag{17}$$

assuming uniform parallel flow far upstream,

$$c(t)=\frac{u_\infty^2}{2} \tag{18}$$

Also, the density-pressure relationship can be expanded as

$$\rho(p)=\rho_\infty+(\frac{d\rho}{dp})(p-p_\infty)+\frac{1}{2}(\frac{d^2\rho}{dp^2})=(p-p_\infty)^2+\ldots \tag{19}$$

therefore,

$$\int_{p_\infty}^{p}\frac{dp}{\rho(p)}=\frac{1}{\rho_\infty}\int_{p_\infty}^{p}\left[1-\frac{p-p_\infty}{p_\infty\gamma}\right]dp\cong\frac{p-p_\infty}{\rho_\infty} \tag{20}$$

Hence, the perturbation pressure can be expressed in terms of the perturbation potential, i.e.,

$$p-p_\infty= -\rho_\infty\left[\frac{\partial \Phi}{\partial t}+u_\infty\frac{\partial \Phi}{\partial x}\right] \tag{21}$$

In pressure coefficient form this becomes

$$C_p=\frac{p-p_\infty}{\frac{1}{2}\rho_\infty u_\infty^2} = -\frac{2}{u_\infty^2}\Phi_t-\frac{2}{u_\infty}\Phi_x \tag{22}$$

For further analysis it is convenient to use non-dimensional quantities. Introducing for the oscillating cascade case the blade chord as the unit of length one has the following non-dimensional coordinates:

$$\bar{x} = \frac{x}{\hat{c}}, \ \bar{y} = \frac{y}{\hat{c}}, \ \bar{t} = \frac{t\,u_\infty}{\hat{c}} \tag{23}$$

Furthermore, introducing the assumption of harmonic time-dependence, one can define a reduced frequency

$$k = \frac{\omega\hat{c}}{u_\infty} \tag{24}$$

where ω is the circular frequency of blade oscillation.

Following previous work by Teipel (Ref. 3) for single blade oscillations the non-dimensional complex amplitude functions

$$U(\bar{x},\bar{y})e^{i\omega t} = \frac{u}{u_\infty} \tag{25}$$

$$V(\bar{x},\bar{y})e^{i\omega t} = \frac{1}{\sqrt{M^2-1}}\ \frac{v}{u_\infty} \tag{26}$$

$$C(\bar{x},\bar{y})e^{i\omega t} = \frac{2}{\gamma-1}\ \frac{1}{M^2}\ \frac{c}{c_\infty} \tag{27}$$

are introduced. Substituting these equations into the continuity and Euler equations (12) and (13) gives

$$\frac{\partial U}{\partial x} + \sqrt{M^2-1}\ \frac{\partial V}{\partial y} + M^2\ \frac{\partial C}{\partial x} + ikM^2C = 0 \tag{28}$$

$$\frac{\partial U}{\partial x} + \frac{\partial C}{\partial x} + ikU = 0 \tag{29}$$

$$\frac{\partial V}{\partial x} + \frac{1}{\sqrt{M^2-1}}\ \frac{\partial C}{\partial y} + ikV = 0 \tag{30}$$

where the overbar on x and y is being dropped for convenience in the subsequent analysis.

Introducing similar non-dimensional variables with distances referred to flexible cylinder length, velocities to free-stream speed and time to cylinder length divided by free-stream speed, one obtains the equation for the non-dimensional perturbation potential, i.e.,

$$(1-M^2)\Phi_{xx} + \Phi_{rr} + \frac{1}{r^2}\Phi_{\theta\theta} - 2M^2\Phi_{xt} - M^2\Phi_{tt} = 0 \qquad (31)$$

subject to the boundary condition

$$\Phi_r\Big|_{r=R} = \frac{\partial h}{\partial t} + \frac{\partial h}{\partial x} \qquad 0\leq x\leq 1$$
$$= 0 \qquad x<0 \qquad (32)$$

If the radial shell deflection is assumed as

$$h(x,\theta,t) = Z(x)\cos n\theta\cdot e^{ikt} \qquad (33)$$

where k is now referred to cylinder length, i.e.,

$$k = \frac{\omega L}{u_\infty} \qquad (34)$$

Then the velocity potential must also have the form,

$$\Phi(x,r,\theta,t) = \phi(x,r)\cos n\theta\cdot e^{ikt} \qquad (35)$$

Substitution of equation (35) into equation (31) then gives

$$(1-M^2)\phi_{xx} + \phi_{rr} + \frac{1}{r}\phi_r - \frac{n^2}{r^2}\phi - 2ikM^2\phi_x + k^2M^2\phi = 0 \qquad (36)$$

subject to

$$\phi_r\Big|_{r=R} = \frac{\partial z}{\partial x} + ikZ \qquad 0\leq x\leq 1$$
$$= 0 \qquad x<0 \qquad (37)$$

The pressure coefficient in terms of the non-dimensional velocity potential is

$$C_p = -2[ik\Phi + \Phi_x] \qquad (38)$$

and the pressure coefficient amplitude is

$$\bar{C}p = -2[ik\phi + \phi_x] \qquad (38a)$$

These are the basic equations used in the subsequent cylindrical shell vibration analysis.

III. THE METHOD OF CHARACTERISTICS

A. CYLINDRICAL SHELL

In Section II, equation (36) was developed as the partial differential equation for $\phi(x,r)$ that requires solution subject to the boundary condition, equation (37). The following developments are based on a previous paper by Platzer, Brix, and Webster, (Ref. 7).

If $M>1$, equation (30) is hyperbolic and has real characteristics, which satisfy the ordinary differential equation

$$(M^2-1)dr^2-dx^2 = 0 \tag{39}$$

An arbitrary function $F(x,r)$ has the following derivatives along the two characteristics:

$$\frac{dF}{ds} = F_x \cdot \frac{dx}{ds} + F_r \frac{dr}{ds} = \frac{\sqrt{M^2-1}}{M} F_x \pm \frac{1}{M} F_r \tag{40}$$

where ds is the differential arc-length along the characteristic $ds^2 = M^2dr^2$ and

$$\frac{dx}{ds} = \frac{\sqrt{M^2-1}}{M} \tag{41}$$

and

$$\frac{dr}{ds} = \pm \frac{1}{M} \tag{42}$$

Letting s_1 be the arc-length of the left-running Mach line and s_2 the arc -length of the right-running Mach line, one obtains for the first derivatives,

$$F_1 = \frac{\sqrt{M^2-1}}{M} F_x + \frac{1}{M} F_r \tag{43}$$

and

$$F_2 = \frac{\sqrt{M^2-1}}{M} F_x - \frac{1}{M} F_r \tag{44}$$

and for the second derivatives

$$-F_{12} = \frac{dF_1}{ds_2} = \frac{1}{M^2}\left[(1-M^2)F_{xx} + F_{rr}\right] = -F_{21} \tag{45}$$

Therefore, equation (30) can be written as

$$\frac{d\phi_1}{ds_2} = \frac{\phi_1-\phi_2}{2rM} - \frac{ikM}{\sqrt{M^2-1}}(\phi_1 + \phi_2) + \left(k^2 - \frac{n^2}{r^2M^2}\right)\phi \tag{46}$$

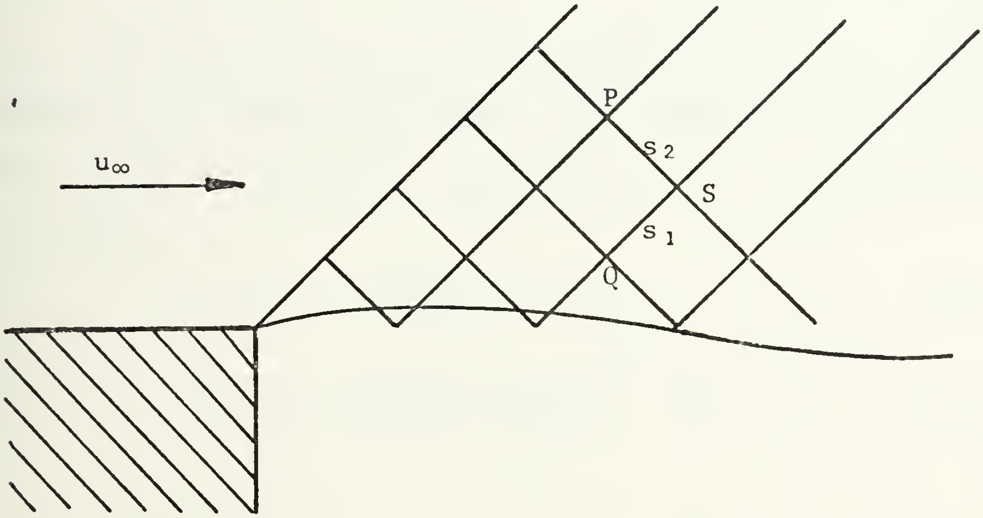


Figure 3. Cylindrical Shell Characteristics Network

The finite difference form of this equation for the points P and S (see Figure 3) on the right-running Mach line is

$$\frac{\phi_1(S) - \phi_1(P)}{\Delta s} = \frac{\phi_1-\phi_2}{2rM} - \frac{ikM}{\sqrt{M^2-1}}(\phi_1+\phi_2) + \left(k^2-\frac{n^2}{r^2M^2}\right)\phi \tag{47}$$

Similarly, for the points Q and S on the left-running Mach line one obtains

$$\frac{\phi_2(S) - \phi_2(Q)}{\Delta s} = \frac{\phi_1-\phi_2}{2rM} - \frac{ikM}{\sqrt{M^2-1}}(\phi_1+\phi_2) + \left(k^2-\frac{n^2}{r^2M^2}\right)\phi \tag{48}$$

Now solving for the values of ϕ_1 and ϕ_2 at S from the known values at P and Q (Figure 3) the entire flow field may be computed point by point. For this purpose the values for ϕ_1 and ϕ_2 in equation (47) are averaged between P and S, i.e.,

$$\phi_1 = \frac{\phi_1(P) + \phi_1(S)}{2} \tag{49}$$

$$\phi_2 = \frac{\phi_2(P) + \phi_2(S)}{2} \tag{50}$$

and

$$\phi(S) = \phi(P) + \frac{1}{2}[\phi_2(P) + \phi_2(S)]\Delta s \tag{51}$$

is obtained by integration along the right-running Mach line.

Similarly, ϕ_1 and ϕ_2 in equation (48) are averaged between Q and S

$$\phi_1 = \frac{\phi_1(Q) + \phi_1(S)}{2} \tag{52}$$

$$\phi_2 = \frac{\phi_2(Q) + \phi_2(S)}{2} \tag{53}$$

and

$$\phi(S) = \phi(Q) + \frac{1}{2}[\phi_1(Q) + \phi_1(S)]\ \Delta s \tag{54}$$

This results in a system of two equations for $\phi_1(S)$ and $\phi_2(S)$, i.e.,

$$A\phi_1(S) + B\phi_2(S) = C \tag{55}$$

$$D\phi_1(S) + E\phi_2(S) = F \tag{56}$$

where

$$A = 1 - \frac{\Delta s}{4Mr(S)} + \frac{ikM\Delta s}{2\sqrt{M^2-1}} \tag{57}$$

$$B = \frac{\Delta s}{4Mr(S)} + \frac{ikM\Delta s}{2\sqrt{M^2-1}} - \frac{\Delta s^2}{4}\left(k^2 - \frac{n^2}{M^2 r^2(S)}\right) \tag{58}$$

$$\begin{aligned} C = {} & \phi_1(P)\left[1 + \frac{\Delta s}{4Mr(P)} - \frac{ikM\Delta s}{2\sqrt{M^2-1}}\right] \\ & + \phi_2(P)\left[- \frac{\Delta s}{4Mr(P)} - \frac{ikM\Delta s}{2\sqrt{M^2-1}} + \frac{\Delta s^2}{4}\left(k^2 - \frac{n^2}{M^2 r^2(S)}\right)\right] \end{aligned} \tag{59}$$

$$D = \frac{\Delta s}{4Mr(S)} - \frac{ikM\Delta s}{2\sqrt{M^2-1}} + \frac{\Delta s^2}{4}\left(k^2 - \frac{n^2}{M^2 r^2(S)}\right)] \tag{60}$$

$$E = -1 - \frac{\Delta s}{4Mr(S)} - \frac{ikM\Delta s}{2\sqrt{M^2-1}} \tag{61}$$

$$\begin{aligned} F = {} & \phi_2(Q)\left[-1 + \frac{\Delta s}{4Mr(Q)} + \frac{ikM\Delta s}{2\sqrt{M^2-1}}\right] \\ & + \phi_1(Q)\left[- \frac{\Delta s}{4r(Q)M} + \frac{ikM\Delta s}{2\sqrt{M^2-1}} - \frac{\Delta s^2}{4}\left(k^2 - \frac{n^2}{M^2 r^2(S)}\right)\right] \\ & + \phi(Q)\left[- \frac{\Delta s}{2}\left(k^2 - \frac{n^2}{r^2(S)M^2}\right) - \frac{\Delta s}{2}\left(k^2 - \frac{n^2}{M^2 r^2(Q)}\right)\right] \end{aligned} \tag{62}$$

The above procedure will suffice for the computation of ϕ and its derivatives at a general flow field point, but needs to be modified to obtain the values on the initial Mach line and along the cylinder surface.

On the initial Mach line (Figure 3) it is known that $\phi=\phi_1=0$. Hence, equation (56) becomes

$$E\phi_2(S) = F \tag{63}$$

where E is known from equation (61) and F simplifies to

$$F = \phi_2(Q)\ [-1 + \frac{\Delta s}{4Mr(Q)} + \frac{ikM\Delta s}{2\sqrt{M^2-1}}] \qquad (64)$$

On the cylinder surface the normal velocity is prescribed, i.e.,

$$\phi_r = \frac{M}{2}(\phi_1-\phi_2) = \frac{\partial Z}{\partial x} + ikZ \qquad (65)$$

whereas equation (55) must hold along the right-running Mach line. The values for ϕ_1 and ϕ_2 on the surface are then found from

$$\phi_1(S) = \frac{C + \frac{2}{M}(\frac{\partial Z}{\partial x} + ikZ)B}{A + B} \qquad (66)$$

and

$$\phi_2(S) = \frac{C - \frac{2}{M}(\frac{\partial Z}{\partial x} + ikZ)B}{A + B} \qquad (67)$$

The pressure distribution over the vibrating cylindrical shell is obtained from the linearized pressure amplitude, equation (38a), and the generalized aerodynamic force Q_{mr} is found from

$$Q_{mr} = \frac{1}{2}\int_0^1 \overline{C}_{p_m}(x)\ \Psi_r(x)dx \qquad (68)$$

where

$$Z(x) = \sum_m A_m\ \Psi_m(x) \qquad (69)$$

and $\overline{C}_{pm}(x)$ is the aerodynamic pressure amplitude due to the deflection $\Psi_m(x)$.

These equations were programmed in Fortran IV. The program description appears in Section IV-A.

B. CASCADE

In Section II equations (28) - (30) were developed as the governing system of partial differential equations for the analysis of the cascade problem. A solution of these equations can be obtained using the method of characteristics. The following development of this method is largely based on the previous work by Teipel (Ref. 3) and Chalkley (Ref. 9), where further details can be found.

Transforming the system into a new coordinate system, $\xi(x,y)$ and $\eta(x,y)$, and considering now a system of three simultaneous equations in unknowns U_ξ, V_ξ, and C_ξ, the equations may be written

$$\xi_x U_\xi + \cot\alpha\ \xi_y V_\xi + M^2 \xi_x C_\xi = -U_\eta \eta_y - \cot\alpha\ V_\eta \eta_y - M^2 C_\eta \eta_x - ikM^2 C \tag{70}$$

$$\xi_x U_\xi + \xi_x C_\xi = -U_\eta \eta_x - C_\eta \eta_x - ikU \tag{71}$$

$$\xi_x V_\xi + \tan\alpha\ \xi_y C_\xi = -V_\eta \eta_x - \tan\alpha\ C_\eta \eta_y - ikV \tag{72}$$

If U_ξ, V_ξ, and C_ξ are made indeterminate across lines of η = constant, the determinant of the matrix of coefficients of the above equations must equal zero, or

$$-\xi_x^3 - \xi_x[\xi_y^{\,2} - M^2\ \xi_x^2] = 0 \tag{73}$$

The solutions to this equation give the three characteristic directions in the physical (x,y) plane:

$$\frac{dy}{dx} = 0 \tag{74}$$

$$\frac{dy}{dx} = \pm \tan\alpha \tag{75}$$

In order to find the compatibility relations of the first derivatives of U, V, and C along the characteristics, equations (70) - (72) are written along lines of ξ = constant, η = constant, and along streamlines, that is, for

$$\mathrm{str}(x,y) = \text{constant}, \ \frac{\partial y}{\partial x} = 0 \tag{76}$$

$$\xi(x,y) = \text{constant}, \ \frac{\partial y}{\partial x} = \frac{1}{\sqrt{M^2-1}} \tag{77}$$

$$\eta(x,y) = \text{constant}, \ \frac{\partial y}{\partial x} = \frac{-1}{\sqrt{M^2-1}} \tag{78}$$

If an arbitrary function of x and y in the equations is denoted by f then, $\frac{\partial f}{\partial x}$ holding str constant becomes

$$\left(\frac{\partial f}{\partial x}\right)_{\mathrm{str}} = \frac{\partial f}{\partial x} \tag{79}$$

Likewise, $\frac{\partial f}{\partial x}$ holding ξ constant becomes,

$$\left(\frac{\partial f}{\partial x}\right)_{\xi} = \frac{\partial f}{\partial x} + \frac{1}{\sqrt{M^2-1}} \frac{\partial f}{\partial y} \tag{80}$$

and, $\frac{\partial f}{\partial x}$ holding η constant becomes

$$\left(\frac{\partial f}{\partial x}\right)_{\eta} = \frac{\partial f}{\partial x} = \frac{1}{\sqrt{M^2-1}} \frac{\partial f}{\partial y} \tag{81}$$

Combining these equations and rewriting,

$$\frac{\partial f}{\partial x} = \left(\frac{\partial f}{\partial x}\right)_{\mathrm{str}} \tag{82}$$

$$\frac{\partial f}{\partial x} = \frac{1}{2}\left[\left(\frac{\partial f}{\partial x}\right)_{\xi} + \left(\frac{\partial f}{\partial x}\right)_{\eta}\right] \tag{83}$$

$$\frac{\partial f}{\partial y} = \frac{1}{2}\sqrt{M^2-1}\ \left[\left(\frac{\partial f}{\partial x}\right)_{\xi} - \left(\frac{\partial f}{\partial x}\right)_{\eta}\right] \tag{84}$$

Substituting these relations into the equations of motion gives

$$\frac{1}{2}\left[\left(\frac{\partial U}{\partial x}\right)_\xi + \left(\frac{\partial U}{\partial x}\right)_\eta\right] + \frac{1}{2}(M^2-1)\left[\left(\frac{\partial V}{\partial x}\right)_\xi - \left(\frac{\partial V}{\partial x}\right)_\eta\right]$$

$$+ \frac{1}{2}M^2\left[\left(\frac{\partial C}{\partial x}\right)_\xi + \left(\frac{\partial C}{\partial x}\right)_\eta\right] + ikM^2C = 0 \qquad (85)$$

$$\frac{1}{2}\left[\left(\frac{\partial U}{\partial x}\right)_\xi + \left(\frac{\partial U}{\partial x}\right)_\eta\right] + \frac{1}{2}\left[\left(\frac{\partial C}{\partial x}\right)_\xi + \left(\frac{\partial C}{\partial x}\right)_\eta\right] + ikU = 0 \qquad (86)$$

$$\left[\left(\frac{\partial V}{\partial x}\right)_\xi + \left(\frac{\partial V}{\partial x}\right)_\eta\right] + \frac{1}{2}\left[\left(\frac{\partial C}{\partial x}\right)_\xi - \left(\frac{\partial C}{\partial x}\right)_\eta\right] + ikV = 0 \qquad (87)$$

Subtracting equation (86) from equation (85) gives an equation that is first added to equation (87) and then subtracted from equation (87). The result is two compatibility relations that must be satisfied along lines of constant ξ and lines of constant η, i.e.,

$$\left(\frac{\partial V}{\partial x}\right)_\xi + \left(\frac{\partial C}{\partial x}\right)_\xi + ik\left[V + \frac{1}{M^2-1}(M^2C-U)\right] = 0 \qquad (88)$$

$$\left(\frac{\partial V}{\partial x}\right)_\eta - \left(\frac{\partial C}{\partial x}\right)_\eta + ik\left[V - \frac{1}{M^2-1}(M^2C-U)\right] = 0 \qquad (89)$$

The third compatibility relation is obtained by considering equation (86) along a streamline

$$\left(\frac{\partial U}{\partial x}\right)_{str} + \left(\frac{\partial C}{\partial x}\right)_{str} + ikU = 0 \qquad (90)$$

If irrotational flow is assumed, the equation of irrotationality

$$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = 0 \qquad (91)$$

may be substituted for the second equation of motion in the system. With the assumption of simple harmonic motion, it may be written in terms of Teipel's amplitude functions,

$$\frac{\partial U}{\partial y} - \sqrt{M^2-1}\,\frac{\partial V}{\partial x} = 0 \qquad (92)$$

The non-dimensional system of equations can then be written as,

$$\frac{\partial U}{\partial x} + \sqrt{M^2-1}\ \frac{\partial V}{\partial y} + M^2\ \frac{\partial C}{\partial x} + ikM^2C = 0 \tag{93}$$

$$\frac{\partial U}{\partial x} + \frac{\partial C}{\partial x} + ikU = 0 \tag{94}$$

$$\frac{\partial U}{\partial y} - \sqrt{M^2-1}\ \frac{\partial V}{\partial x} = 0 \tag{95}$$

By transforming equations (93) - (95) to the arbitrary coordinate system, ξ and η, the system can be considered one of simultaneous equations in U_ξ, V_ξ, and C_ξ. By making any of these derivatives indeterminate, the results of setting the determinant of the matrix of their coefficients equal to zero are equations (74) and (75), the characteristic directions.

By writing the equations of motion, as before, along lines of ξ = constant and η = constant and performing the same algebraic manipulations, the two compatibility relations can be determined:

$$\left(\frac{\partial U}{\partial x}\right)_\xi - \left(\frac{\partial V}{\partial x}\right)_\xi + ik\ \frac{M^2}{M^2-1}\ (U-C) = 0 \tag{96}$$

$$\left(\frac{\partial U}{\partial x}\right)_\eta + \left(\frac{\partial V}{\partial x}\right)_\eta + ik\ \frac{M^2}{M^2-1}\ (U-C) = 0 \tag{97}$$

The third compatibility relation is, as before,

$$\left(\frac{\partial U}{\partial x}\right)_{str} + \left(\frac{\partial C}{\partial x}\right)_{str} + ikU = 0 \tag{90}$$

The equations of motion are now reduced to a system containing only derivatives with respect to x. It is, thus, possible to solve these equations using finite differences in the following manner: separating the real from the imaginary parts, the equations can be written,

$$\left(\frac{\partial U_R}{\partial x}\right)_{str} + \left(\frac{\partial C_R}{\partial x}\right)_{str} - kU_I = 0 \qquad (98)$$

$$\left(\frac{\partial U_I}{\partial x}\right)_{str} + \left(\frac{\partial C_I}{\partial x}\right)_{str} + kU_R = 0 \qquad (99)$$

$$\left(\frac{\partial U_R}{\partial x}\right)_{\xi} - \left(\frac{\partial V_R}{\partial x}\right)_{\xi} - k\,\frac{M^2}{M^2-1}\,(U_I - C_I) = 0 \qquad (100)$$

$$\left(\frac{\partial U_I}{\partial x}\right)_{\xi} - \left(\frac{\partial V_I}{\partial x}\right)_{\xi} + k\,\frac{M^2}{M^2-1}\,(U_R - C_R) = 0 \qquad (101)$$

$$\left(\frac{\partial U_R}{\partial x}\right)_{\eta} + \left(\frac{\partial V_R}{\partial x}\right)_{\eta} - k\,\frac{M^2}{M^2-1}\,(U_I - C_I) = 0 \qquad (102)$$

$$\left(\frac{\partial U_I}{\partial x}\right)_{\eta} + \left(\frac{\partial V_I}{\partial x}\right)_{\eta} + k\,\frac{M^2}{M^2-1}\,(U_R - C_R) = 0 \qquad (103)$$

To write these equations in finite difference form the computational molecule shown in Figure 4 will be used. With the velocity values all known at P_{11}, P_{12}, and P_{21}, all the values at P_{22} can be calculated. The distance Δx is arbitrary.

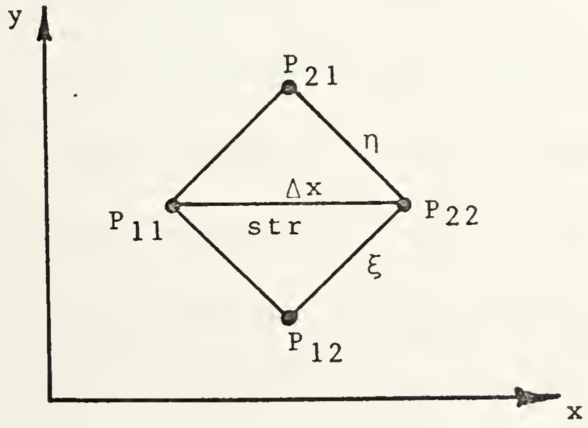


Figure 4. General Flow Field Molecule

If F is an arbitrary flow quantity, its partial derivatives can be written as

$$\left(\frac{\partial F}{\partial x}\right)_{str} = \frac{F_{22} - F_{11}}{\Delta x} \tag{104}$$

$$\left(\frac{\partial F}{\partial x}\right)_{\xi} = \frac{F_{22} - F_{12}}{\frac{1}{2}\Delta x} \tag{105}$$

$$\left(\frac{\partial F}{\partial x}\right)_{\eta} = \frac{F_{22} - F_{21}}{\frac{1}{2}\Delta x} \tag{106}$$

The values of the flow quantities in the equations are averages, such that

$$(F)_{str} = \frac{1}{2}(F_{11} + F_{22}) \tag{107}$$

$$(F)_{\xi} = \frac{1}{2}(F_{12} + F_{22}) \tag{108}$$

$$(F)_{\eta} = \frac{1}{2}(F_{21} + F_{22}) \tag{109}$$

With these relations substituted into equations (98) - (103), the system becomes after some algebraic manipulation,

$$U_{22R} + C_{22R} - A_I U_{22I} = K_{12R} \tag{110}$$

$$U_{22I} + C_{22I} + A_I U_{22R} = K_{12I} \tag{111}$$

$$U_{22R} - V_{22R} + B_I(C_{22I} - U_{22I}) = K_{34R} \tag{112}$$

$$U_{22I} - V_{22I} + B_I(U_{22R} - C_{22R}) = K_{34I} \tag{113}$$

$$U_{22R} + V_{22R} + B_I(C_{22I} - U_{22I}) = K_{56R} \tag{114}$$

$$U_{22I} + V_{22I} + B_I(U_{22R} - C_{22R}) = K_{56I} \tag{115}$$

where

$$K_{12R} = U_{11R} + C_{11R} + A_I U_{11I} \tag{116}$$

$$K_{12I} = U_{11I} + C_{11I} - A_I U_{11R} \tag{117}$$

$$K_{34R} = U_{12R} - V_{12R} + B_I(U_{12I} - C_{12I}) \tag{118}$$

$$K_{34I} = U_{12I} - V_{12I} - B_I(U_{12R} - C_{12R}) \tag{119}$$

$$K_{56R} = U_{21R} + V_{21R} + B_I(U_{21I} - C_{21I}) \tag{120}$$

$$K_{56I} = U_{21I} + V_{21I} - B_I(U_{21R} - C_{21R}) \tag{121}$$

and

$$A_I = \frac{1}{2} k\Delta x \tag{122}$$

$$B_I = \frac{1}{4} k \frac{M^2}{M^2-1} \Delta x \tag{123}$$

As shown in Teipel (Ref. 3), solving these equations gives,

$$U_{22R} = \frac{(1-A_IB_I)[\frac{1}{2}(K_{34R}+K_{56R})-B_IK_{12I}]+2B_I[\frac{1}{2}(K_{34I}+K_{56I})+B_IK_{12R}]}{(1-A_IB_I)^2 + (2B_I)^2} \tag{124}$$

$$U_{22I} = \frac{(1-A_IB_I)[\frac{1}{2}(K_{34I}+K_{56I})+B_IK_{12R}]-2B_I[\frac{1}{2}(K_{34R}+K_{56R})-B_IK_{12I}]}{(1-A_IB_I)^2 + (2B_I)^2} \tag{125}$$

$$V_{22R} = \frac{1}{2} (K_{56R} - K_{34R}) \tag{126}$$

$$V_{22I} = \frac{1}{2} (K_{56I} - K_{34I}) \tag{127}$$

$$C_{22R} = K_{12R} - U_{22R} + A_IU_{22I} \tag{128}$$

$$C_{22I} = K_{12I} - U_{22I} - A_IU_{22R} \tag{129}$$

These are the finite difference equations for a point in the general flow field.

At a point on the upper surface of a cascade blade, the normal velocity V_{22} is prescribed by the movement of the blade.

This velocity was given in the problem formulation. Here the computational molecule is shown in Figure 5.

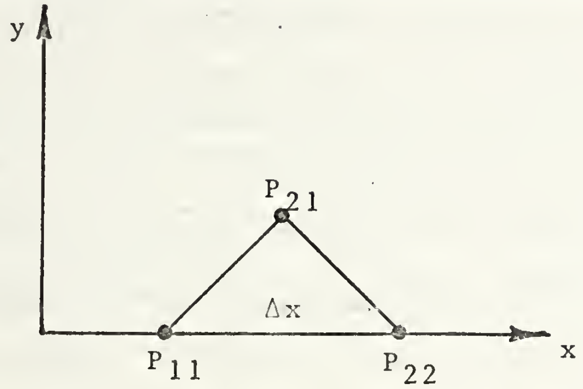


Figure 5. Upper Surface Molecule

The applicable equations are,

$$U_{22R} + C_{22R} - A_I U_{22I} = K_{12R} \tag{130}$$

$$U_{22I} + C_{22I} + A_I U_{22R} = K_{12I} \tag{131}$$

$$U_{22R} - B_I U_{22I} + B_I C_{22I} = K_{56R} - K_{34R} \tag{132}$$

$$U_{22I} + B_I U_{22R} - B_I C_{22R} = K_{56I} - K_{34I} \tag{133}$$

where K_{12} and K_{56} are as before, but,

$$K_{34R} = V_{22R} \tag{134}$$

$$K_{34I} = V_{22I} \tag{135}$$

as given by the boundary conditions. Solving for the flow quantities gives

$$U_{22R} = \frac{(1-A_I B_I)[K_{56R}-K_{34R}-B_I K_{12I}]+2B_I[K_{56I}-K_{34I}+B_I K_{12R}]}{(1-A_I B_I)^2 + (2B_I)^2} \tag{136}$$

$$U_{22I} = \frac{(1-A_I B_I)[K_{56I}-K_{34I}+B_I K_{12R}]-2B_I[K_{56R}-K_{34R}-B_I K_{12I}]}{(1-A_I B_I)^2 + (2B_I)^2} \tag{137}$$

$$C_{22R} = K_{12R} - U_{22R} + A_I U_{22I} \tag{138}$$

$$C_{22I} = K_{12I} - U_{22I} - A_I U_{22R} \tag{139}$$

and V_{22R} and V_{22I} are known from the boundary conditions. These are the finite difference equations for the flow velocities on the upper surface of a cascade blade.

Likewise, at the lower surface of a cascade blade, the normal velocity, V_{22} is prescribed by the movement of the blade, and was given in the problem formulation. The computational molecule, different from that at the upper surface of the blade, is shown in Figure 6.

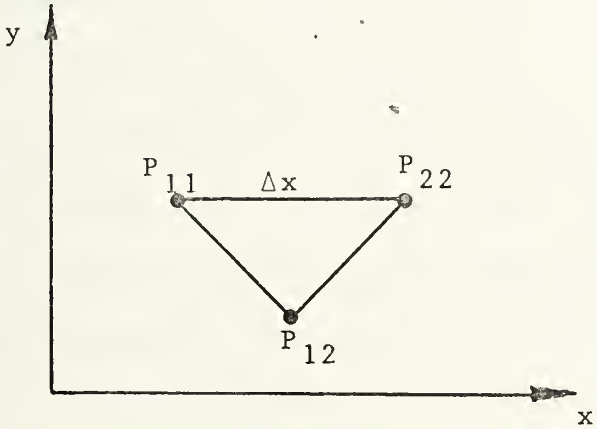


Figure 6. Lower Surface Molecule

Here, the applicable equations are,

$$U_{22R} + C_{22R} - A_I U_{22I} = K_{12R} \tag{140}$$

$$U_{22I} + C_{22I} + A_I U_{22R} = K_{12I} \tag{141}$$

$$U_{22R} - B_I U_{22I} + B_I C_{22I} = K_{56R} + K_{34R} \tag{142}$$

$$U_{22I} + B_I U_{22R} - B_I C_{22R} = K_{56I} + K_{34I} \tag{143}$$

where K_{12} and K_{34} are as originally stated, but,

$$K_{56R} = V_{22R} \tag{144}$$

$$K_{56I} = V_{22I} \tag{145}$$

as given by the boundary conditions. Thus, the finite difference equations for the flow velocities at the lower surface of a cascade blade are,

$$U_{22R} = \frac{(1-A_I B_I)[K_{56R}+K_{34R}-B_I K_{12I}]+2B_I[K_{56I}+K_{34I}+B_I K_{12R}]}{(1-A_I B_I)^2 + (2B_I)^2} \qquad (146)$$

$$U_{22I} = \frac{(1-A_I B_I)[K_{56I}+K_{34I}+B_I K_{12R}]-2B_I[K_{56R}+K_{34R}-B_I K_{12I}]}{(1-A_I B_I)^2 + (2B_I)^2} \qquad (147)$$

$$C_{22R} = K_{12R} - U_{22R} + A_I U_{22I} \qquad (148)$$

$$C_{22I} = K_{12I} - U_{22I} - A_I U_{22R} \qquad (149)$$

where V_{22R} and V_{22I} are given from the boundary condition.

To obtain the conditions on the initial left-running Mach line, assume that P_{11} and P_{21} as shown in Figure 4 are just in the free-stream and then let Δx shrink to zero. Since $K_{12} = K_{56} = 0$ and A_I and B_I go to zero as Δx goes to zero, the initial finite difference equations for ξ=constant are,

$$U_{22} + C_{22} = 0 \qquad (150)$$

$$U_{22} - V_{22} + V_{12} = 0 \qquad (151)$$

$$U_{22} + V_{22} = 0 \qquad (152)$$

thus,

$$U_{22} = - V_{22} = - C_{22} \qquad (153)$$

Equation (96) can be written,

$$\left(\frac{\partial U}{\partial x}\right)_\xi = - ikU \qquad (154)$$

Integrating gives,

$$U = U|_{x=0} e^{-ik\frac{M^2}{M^2-1}x} \quad (155)$$

thus,

$$U_{22R} = -V_{22R}(0)\cos\left(k\frac{M^2}{M^2-1}x\right) - V_{22I}(0)\sin\left(k\frac{M^2}{M^2-1}x\right) \quad (156)$$

$$U_{22I} = -V_{22I}(0)\cos\left(k\frac{M^2}{M^2-1}x\right) + V_{22R}(0)\sin\left(k\frac{M^2}{M^2-1}x\right) \quad (157)$$

$$V_{22R} = -U_{22R} \quad (158)$$

$$V_{22I} = -U_{22I} \quad (159)$$

$$C_{22R} = -U_{22R} \quad (160)$$

$$C_{22I} = -U_{22I} \quad (161)$$

For the initial right-running Mach line, P_{11} and P_{12} of Figure 4 are assumed to be just in the free-stream and Δx is brought to zero. Since $K_{12} = K_{34} = 0$, and $A_I = B_I = 0$,

$$U_{22} + C_{22} = 0 \quad (162)$$

$$U_{22} - V_{22} = 0 \quad (163)$$

$$U_{22} - U_{21} + V_{22} = 0 \quad (164)$$

thus,

$$U_{22} = V_{22} = -C_{22} \quad (165)$$

Equation (97) can then be written,

$$\left(\frac{\partial U}{\partial x}\right)_\eta = -ikU \quad (166)$$

Integrating gives,

$$U = V(0)e^{-ik\frac{M^2}{M^2-1}x} \quad (167)$$

Thus,

$$U_{22R} = V_{22R}(0)\cos\left(k \frac{M^2}{M^2-1} x\right)+V_{22I}(0)\sin\left(k \frac{M^2}{M^2-1} x\right) \quad (168)$$

$$U_{22I} = V_{22I}(0)\cos\left(k \frac{M^2}{M^2-1} x\right)-V_{22R}(0)\sin\left(k \frac{M^2}{M^2-1} x\right) \quad (169)$$

$$V_{22R} = U_{22R} \quad (170)$$

$$C_{22R} = -U_{22R} \quad (171)$$

$$V_{22I} = U_{22I} \quad (172)$$

$$C_{22I} = -U_{22I} \quad (173)$$

At the first position encountered on a blade other than the first blade there are two grid points for which the flow field quantities must be computed; one on the upper surface and one on the lower surface of the blade.

As in Figure 7 the blade is assumed to protrude a very small distance past P_{22}, thus necessitating the computation of the upper and lower flow velocities at this point. In this case, the equations developed earlier for an upper surface grid point are used for P_{22U} and those developed for a lower surface grid point are used at P_{22L}.

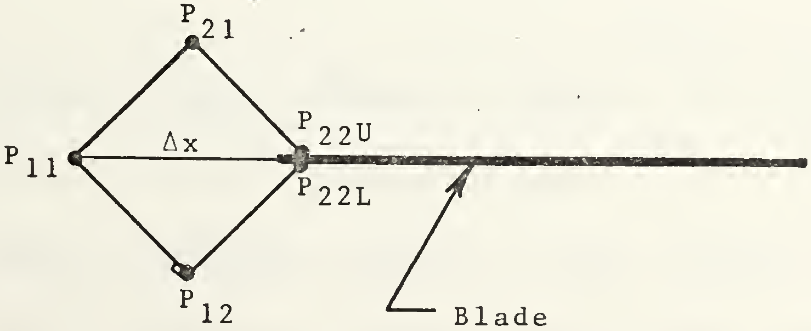


Figure 7. New Blade Molecule

At a position just aft of a blade there are also two grid points to be accounted for; one above and one below the slip plane in the wake. As in Figure 8 the point P_{22} is assumed to be a very small distance aft of the blade's trailing edge.

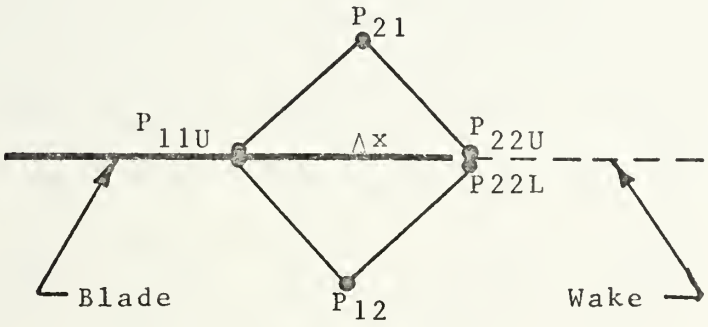


Figure 8. Wake Molecule

At this position the equations for an upper blade grid point apply to P_{22U} and those for a lower grid point apply to P_{22L}, i.e.,

$$U_{22RU} + C_{22RU} - A_I U_{22IU} = K_{12RU} \tag{174}$$

$$U_{22IU} + C_{22IU} + A_I U_{22RU} = K_{12IU} \tag{175}$$

$$U_{22RU} + B_I (C_{22IU} - U_{22IU}) = K_{56R} - V_{22RU} \tag{176}$$

$$U_{22IU} + B_I (U_{22RU} - C_{22RU}) - K_{56I} - V_{22IU} \tag{177}$$

and

$$U_{22RL} + C_{22RL} - A_I U_{22IL} = K_{12RL} \tag{178}$$

$$U_{22IL} + C_{22IL} + A_I U_{22RL} = K_{12IL} \tag{179}$$

$$U_{22RL} + B_I (C_{22IL} - U_{22IL}) = V_{22RL} + K_{34R} \tag{180}$$

$$U_{22IL} + B_I (U_{22RL} - C_{22RL}) = V_{22IL} + K_{34I} \tag{181}$$

where

$$K_{12RU} = U_{11RU} + C_{11RU} + A_I U_{11IU} \tag{182}$$

$$K_{12IU} = U_{11IU} + C_{11IU} - A_I U_{11RU} \tag{183}$$

$$K_{12RL} = U_{11RL} + C_{11RL} + A_I U_{11IL} \tag{184}$$

$$K_{12IL} = U_{11IL} + C_{11IL} - A_I U_{11RL} \tag{185}$$

$$K_{34R} = U_{12R} - V_{12R} + B_I(U_{12I} - C_{12I}) \tag{186}$$

$$K_{34I} = U_{12I} - V_{12I} - B_I(U_{12R} - C_{12R}) \tag{187}$$

$$K_{56R} = U_{21R} + V_{21R} + B_I(U_{21I} - C_{21I}) \tag{188}$$

$$K_{56I} = U_{21I} + V_{21I} - B_I(U_{21R} - C_{21R}) \tag{189}$$

Equations (174) - (181) form a system of eight equations with twelve unknowns. However, since it is known that the wake cannot support a pressure difference and that the normal velocities on either side of the wake must be equivalent, i.e.,

$$V_{22RU} = V_{22RL} \equiv V_{22R} \tag{190}$$

$$V_{22IU} = V_{22IL} \equiv V_{22I} \tag{191}$$

$$C_{22RU} = C_{22RL} \equiv C_{22R} \tag{192}$$

$$C_{22IU} = C_{22IL} \equiv C_{22I} \tag{193}$$

we can eliminate four of the unknowns and achieve a determinate system of equations. Equations (174) - (181) become

$$U_{22RU} + C_{22R} - A_I U_{22IU} = K_{12RU} \tag{194}$$

$$U_{22IU} + C_{22I} + A_I U_{22RU} = K_{12IU} \tag{195}$$

$$U_{22RU} + V_{22R} + B_I(C_{22I} - U_{22IU}) = K_{56R} \tag{196}$$

$$U_{22IU} + V_{22I} + B_I(U_{22RU} - C_{22R}) = K_{56I} \tag{197}$$

$$U_{22RL} + C_{22R} - A_I U_{22IL} = K_{12RL} \qquad (198)$$

$$U_{22IL} + C_{22I} + A_I U_{22RL} = K_{12IL} \qquad (199)$$

$$U_{22RL} - V_{22R} + B_I(C_{22I} - U_{22IL}) = K_{34R} \qquad (200)$$

$$U_{22IL} - V_{22I} + B_I(U_{22RL} - C_{22R}) = K_{34I} \qquad (201)$$

The pressure distributions along the surfaces of the blades can be determined from equation (11). Dividing both sides of this equation by p_∞ and using Teipel's amplitude function for c, the equation can be written,

$$P(x,y) = \gamma M^2 C(x,y)\Big|_{y=0^+,\ y=0^-} \qquad (202)$$

where

$$P(x,y)e^{i\omega t} = \frac{p-p_\infty}{p_\infty} \qquad (203)$$

Thus the non-dimensional pressure distributions along the upper and lower surfaces of the blades in a cascade can be computed in a point by point calculation of U, V, and C along lines of ξ=constant and η=constant.

These equations were programmed in Fortran IV. The program description appears in Section IV-B.

C. THE LINEARIZED METHOD OF CHARACTERISTICS OF SAUER AND HEINZ

Sauer and Heinz (Ref. 11) developed a linearized method of characteristics for supersonic flow past bodies of revolution at zero angle of attack. This method also is derived from the linearized potential equation for steady supersonic flow, i.e.,

$$(M^2-1)\phi_{xx} - \phi_{rr} - \frac{1}{r}\phi_r = 0 \qquad (204)$$

Sauer and Heinz showed that the compatibility relations can be written as

$$\frac{\partial (rv)}{\partial \xi} = -\sqrt{M^2-1}\; r \frac{\partial u}{\partial \xi} \qquad (205)$$

and

$$\frac{\partial (rv)}{\partial \eta} = \sqrt{M^2-1}\; r \frac{\partial u}{\partial \eta} \qquad (206)$$

where

$$u = \frac{\partial \phi}{\partial x} \qquad (207)$$

$$v = \frac{\partial \phi}{\partial r} \qquad (208)$$

and

$$\xi = x - \sqrt{M^2-1}\; r \qquad (209)$$

$$\eta = x + \sqrt{M^2-1}\; r \qquad (210)$$

are the characteristic coordinates as explained previously.

Writing this equation in finite difference form one obtains for u and v at point 4 (Figure 9) using the known values in points 2 and 3,

$$U_4 = \frac{(rv)_3-(rv)_2+\sqrt{M^2-1}(r_\eta u_3+r_\xi u_2}{\sqrt{M^2-1}\;(r_\xi+r_\eta)} \qquad (211)$$

$$(vr)_4 = \frac{r_\xi (rv)_3+r_\eta (rv)_2-\sqrt{M^2-1}\; r_\xi r_\eta (u_2-u_3)}{r_\xi + r_\eta} \qquad (212)$$

where r_η and r_ξ are the average r coordinates between 3 and 4 and between 2 and 4 respectively. Computing the flow properties at the body using the exact flow tangency condition leads to

$$u_5 = \frac{\sqrt{M^2-1}\ r_\eta u_4 + (rv)_4 - r_5\ (\frac{dh}{dx})_5}{\sqrt{M^2-1}\ r_\eta\ + r_5 (\frac{dh}{dx})_5} \tag{213}$$

$$v_5 = (\frac{dh}{dx})_5 (1+u_5) \tag{214}$$

where r = h(x) describes the body contour.

However, in order to apply the Sauer-Heinz procedure to the case of an infinitely long cylinder with a wavy surface between $0<x<1$ a different starting procedure is needed. It is easily seen from Figure 9 that

$$r_2 = \frac{h_0}{2}\ (\frac{1}{\sqrt{M^2-1}\ \tan\ \delta_0}\ + 1) \tag{215}$$

realizing that

$$r_\eta = \frac{R_1 + R_2}{2} \tag{216}$$

where

$$R_1 = R_0 + r_1 \tag{217}$$

and

$$R_2 = R_0 + h_0 \tag{218}$$

one obtains

$$r_\eta = R + \frac{h_0}{4}\ (3 + \frac{1}{\sqrt{M^2-1}\ \tan\ \delta_0}) \tag{219}$$

Since there is no disturbance upstream of the leading edge Mach line, one obtains from equation (213)

$$U_2 = \frac{-\ (R+h_0)\ \tan\ \delta_0}{\sqrt{M^2-1}\ r_\eta + (R+h_0)\ \tan\ \delta_0} \tag{220}$$

where the point 2 is the first body point.

Finally, the pressure coefficient is obtained from

$$C_p - 2u - v^2 + u^2 \ (M^2-1) \qquad (221)$$

The program listing for these equations is given in Appendix I.

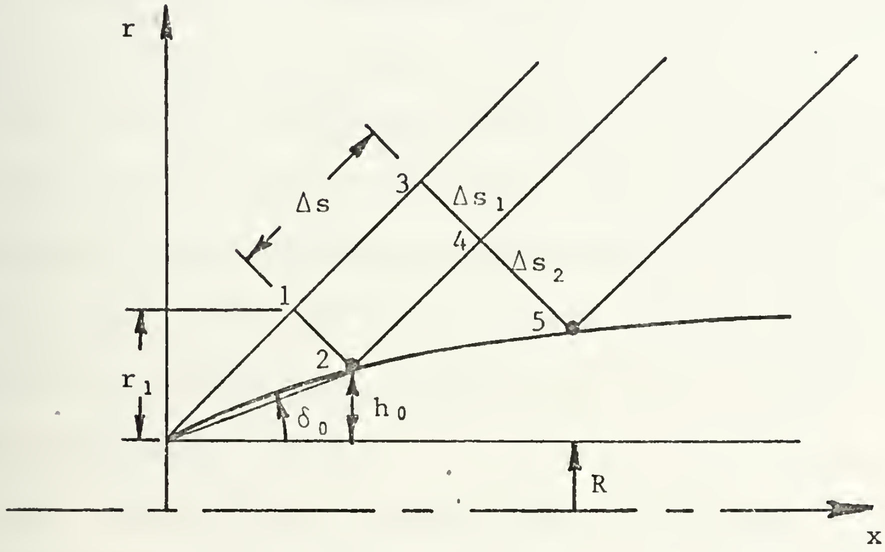


Figure 9. Sauer-Heinz Characteristic Network

IV. COMPUTATIONAL PROCEDURE

A. CYLINDRICAL SHELL COMPUTER PROGRAM

This program calculates the pressure distribution over a harmonically vibrating cylindrical shell. It is valid only for a cylinder in a supersonic flow field vibrating with small amplitude vibrations.

The program calculates the flow field properties adjacent to the cylinder using the method of characteristics finite difference equations developed previously in Section III. The pressure distribution is then integrated using the trapezoidal rule according to equation (68) to obtain the generalized aerodynamic force.

The computational diagram shown in Figure 3 is used to calculate the flow field quantities at S in the general flow field and on the cylinder surface. The distance Δs shown in the figure is determined by

$$\Delta s = \frac{L \cos \alpha}{\nu} \qquad (222)$$

where ν is the grid fineness ratio. The grid fineness ratio is an arbitrary input variable into the program and is equal to one less than the total number of grid points on the cylinder surface.

Input parameters are entered into the program on two data cards. The first card contains the date, e.g., 11 APR 1973, in the first twelve columns. The second card contains the following input data:

SPACE	DATA
1-20	Free-stream Mach number, M
21-30	Radius to length ratio, R
31-40	Reduced frequency, k
41-45	Axial mode number, m
46-50	r, as defined in equation (68) for the generalized aerodynamic force.
51-55	Circumferential mode number, n
56-60	Grid fineness ratio, ν

where the first three quantities are floating point numbers and the rest are integers. The definitions of these parameters are given again in the alphabetical listing of all program variables in Appendix A.

The program has five subroutines. They are as follows:

COMPXR	Computes the x and r position of the given grid point.
MACHLN	Computes the flow quantities along the initial left-running Mach line at the given grid point.
GENFPT	Computes the flow quantities at a general flow field grid point.
PANPNT	Computes the flow quantities on the cylinder surface at the given grid point.
INTEG	Computes the generalized aerodynamic force.

Flow quantities along the initial left-running Mach line are computed using the equations derived in the method of characteristics for initial conditions. Those quantities on the cylinder surface are computed using the equations derived

for the boundary condition. The values of the velocity potential and its derivatives are derived as a result of the analysis of the flow tangency condition and have been given previously. The procedure starts at the initial Mach line and continues down successive right-running Mach lines. The pressure coefficient is computed when the cylinder surface is met and stored in an array for later use in the integration of the generalized aerodynamic force.

A flow diagram of this program is given in Appendix B.

B. OSCILLATING CASCADE PROGRAM

This program calculates the pressure distribution over the upper and lower surfaces of the blades of a subsonic leading edge locus cascade which is oscillating in pitch at arbitrary frequency and interblade phase angle. It is valid only for a cascade in a supersonic flow field vibrating with small amplitude vibrations. The program calculates the flow field quantities within the area governed by the aft-running Mach lines emanating from the leading edge of the first blade and the forward-running Mach lines emanating from the trailing edge of the last blade of the cascade (see Figure 10). These flow field properties are calculated using the method of characteristics finite difference equations developed previously in Section III-B.

The computational molecules shown in Figures 4, 5, 6, 7, and 8 are used to calculate the flow field properties in the general flow field, upper surfaces, lower surfaces, leading edge of a staggered blade, and the wake aft of a blade,

respectively. The distance Δx shown in the figures is determined by

$$\Delta x = \frac{2d}{\nu} \cot \alpha \tag{223}$$

where ν is the grid fineness ratio. This parameter is an arbitrary input variable into the program and is equal to one less than the total number of grid points on any Mach line that runs from one blade to the next.

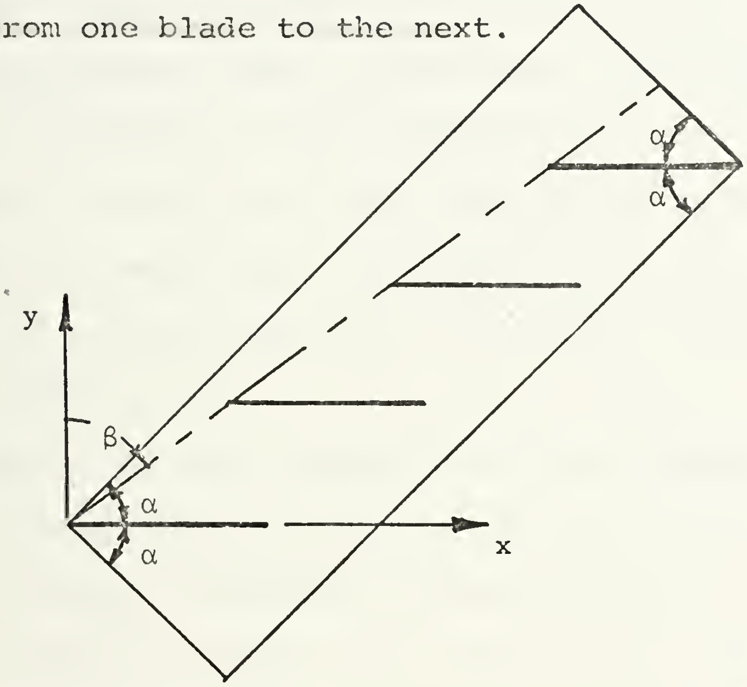


Figure 10. Cascade Computational Area

Input parameters are entered into the program by means of the NAMELIST option of Fortran. In using this option, input parameters are combined in a titled list and then referred to in the program by this title (NAM1). The real advantage of using NAMELIST is that each data card need only contain the value of one input variable while the format of the data card is such that the variable's name is punched on the card as well as its value. In this way, when a different value for an input variable is desired, rather than changing the

entire data deck, only one easily identifiable data card need be changed.

The format for a NAMELIST data deck is as follows: The first column in each card of the data deck is left blank. The first card starts in the second column with the symbol & (ampersand) followed immediately by the title of the NAMELIST (here, NAM1). The next cards list the variables and their input values, one per card, in any order. The format here, starting in the second column and followed by a comma, is: variable name = value. The last card in the data deck and the signal to the computer that the NAMELIST has ended again starts in the second column with the symbol & immediately followed by END.

The value of an input variable may be written in any format as long as integer variables are written without decimal points and real variables are written with a decimal point. Blanks are taken as zeroes, but a comma must appear somewhere between a desired input value and the next data card. The date of the run is entered on a data card in the first twelve spaces. This data card is inserted prior to the NAMELIST dataset.

To illustrate the input procedure, the following example is proposed:

$\nu = 100$ $\quad d = 0.8$

$M = \sqrt{2}$ $\quad \beta = 55°$

$k = 0.75$ $\quad \delta = 45°$

$x_0 = 0.5$

5 blades

The data cards needed are as follows:

CARD	FORMAT	DESCRIPTION
1	22 JUNE 1973	Date
2	&NAM1	NAMELIST title
3	NGRDFN=100,	Grid fineness ratio, must be an even integer, subject to the constraint of equation (227).
4	FSTRMN=1.414214,	Free-stream Mach number
5	REDFRQ=0.80,	Reduced frequency
6	XSUBO=0.50,	Elastic axis position
7	TNWDST=0.80,	Vertical distance between blades.
8	STGANG=55.0,	Complimentary stagger angle, $\tan\beta > \cot\alpha$.
9	FAZE=45.0,	Interblade phase angle
10	NUMBLD=5,	Number of blades
11	&END	End of NAMELIST

The definitions of these parameters are given again in the alphabetical listing of all program variables in Appendix C.

The program has nine subroutines. They are as follows:

INPUT	Reads in all input data and sets up the finite difference grid.
INTIAL	Initializes all the flow quantities at the leading edge of the first blade and initializes most of the logic variables.
MACHLN	Computes the flow quantities along the initial left and right-running Mach lines at the given grid point.
TOP	Computes the flow quantities on the upper surface of a blade at the given grid point.

GENFPT	Computes the flow quantities at the given general flow field point.
BOTTOM	Computes the flow quantities on the lower surface of a blade at the given grid point.
NEWBLD	Computes the flow quantities at the leading edge of all staggered blades.
WAKE	Computes the flow quantities in the wake aft of each blade at the given grid point. This subroutine uses IBM subroutine SIMQ for the solution of its simultaneous equations.
COMPXY	Computes the x and y position of the given grid point.

In INPUT the grid is determined using the grid fineness ratio, blade spacing, number of blades, and the free-stream Mach number. The value of Δx as shown in equation (223) is obtained. The stagger angle is then adjusted such that the distance the upper blades are staggered back is the nearest multiple number of Δx increments. The "compatible stagger angle" is then printed out. The blades may be staggered such that the first staggered blade is at least one Δx increment aft of the initial left-running Mach line, but no further forward, or,

$$d \cdot (\tan \beta - \cot \alpha) \geq \Delta x \tag{224}$$

Flow quantities along the initial left and right-running Mach lines are computed using the equations derived in the method of characteristics, Section B, for initial conditions. Those flow quantities along the blades' surfaces are computed using the equations derived for the appropriate boundary condition. The equations for the normal velocities are derived as a result of the analysis of the flow tangency condition and have been given previously. The flow quantities

in the wake aft of each blade are computed using the set of simultaneous equations derived as a result of the assumptions made about the flow quantities on either side of the wake and are given in the Method of Characteristics, Section III-B.

The procedure starts at the leading edge of the first blade and computes the flow quantities at grid points along the initial right-running Mach line. It then jumps back to the initial left-running Mach line and computes the flow quantities along the next right-running Mach line, using the applicable subroutines at the appropriate grid points. Only flow quantities on the Mach line presently being computed and the Mach line which preceeded it are kept in storage at any one time. Flow quantities at successive grid points are computed in like manner until the trailing edge of the last blade of the cascade is met.

A flow diagram of this program is given in Appendix D.

C. SAUER-HEINZ COMPUTER PROGRAM

This program computes the pressure distribution over a cylindrical shell or body of revolution. It is written so that the shape of the body or cylindrical surface may be easily changed. It is valid only for non-oscillating bodies or shells in a supersonic flow field at zero angle of attack. The following discussion applies to the cylindrical shell.

The program calculates the flow field adjacent the cylinder using the method of characteristics finite difference equations developed previously in Section III-C.

The computational diagram shown in Figure 9 is used to compute the flow field properties at 4 in the general flow field and at 5 on the cylinder surface. The distance Δs shown in the figure is determined by,

$$\Delta s = \frac{\cos \alpha}{\nu} \qquad (225)$$

where ν is the grid fineness ratio. The grid fineness ratio is an arbitrary input variable into the program and is equal to one less than the number of grid points on the initial Mach line. The length Δs_n, n=1,2,3... is determined by the intersection of the right-running Mach lines with the surface.

Input parameters are entered into the program on one data card. This card contains the following input data:

SPACES	DATA
1-10	Free-stream Mach number, M
11-20	Maximum amplitude of the axial mode, h_0
21-30	Cylinder radius to length ratio, R.
31-40	Axial mode number, m
41-50	Grid fineness ratio, ν

where the first three quantities are floating point numbers and the last two are integers. The definitions of these quantities are given again in the alphabetical listing of all program variables in Appendix E.

The program has three subroutines. They are as follows:

COMPXR	Computes the x and r position of the grid point and determines the position of the intersection of the current right-running Mach line with the body.
GENFPT	Computes the flow quantities at a general flow field grid point.

SURFAC	Computes the flow quantities at the given surface point.

Flow quantities on the initial left-running Mach line are assumed to be zero. Those quantities on the surface are computed using the appropriate equations developed in Section III-C, for surface grid points.

The procedure starts at the initial Mach line and continues down successive right-running Mach lines. The pressure coefficient is computed when the surface is met.

A flow diagram of this program is given in Appendix F.

V. RESULTS AND COMPARISONS

A. CYLINDRICAL SHELL

Pressure distributions and generalized aerodynamic forces were computed for sinusoidal deflection modes, i.e.,

$$\Psi_m(x) = \sin(m\pi x) \qquad m=1,2,... \qquad (226)$$

Figures 11 through 16 show results for the generalized aerodynamic forces Q_{11} and Q_{12} as a function of circumferential mode number for two radius to length ratios, $r=0.5$ and $r=1.0$, at a Mach number of $\sqrt{2}$ and reduced frequencies of zero and one. Also shown is the comparison with two previous solutions by Dowell and Widnall (Ref. 2) and by Matsuzaki and Kobayashi (Ref. 5). The Dowell and Widnall solution is also based on the complete linearized unsteady potential equation, but uses Laplace transform techniques for its solution. As can be seen, agreement between this solution and the method of characteristics solution is excellent. For the sake of interest, a recent approximate solution by Matsuzaki and Kobayashi, valid only for large values of the circumferential mode number is also shown.

Figures 17 through 24 show the generalized aerodynamic forces Q_{11R} and Q_{12R} as a function of radius to length ratio again for a Mach number of $\sqrt{2}$, zero reduced frequency, and circumferential mode numbers $n=0,1,2$, and 3. The main purpose of these figures is to show that the infinite cylinder theory is a poor approximation for small radius to length ratios, but approaches the exact theory for large values of R.

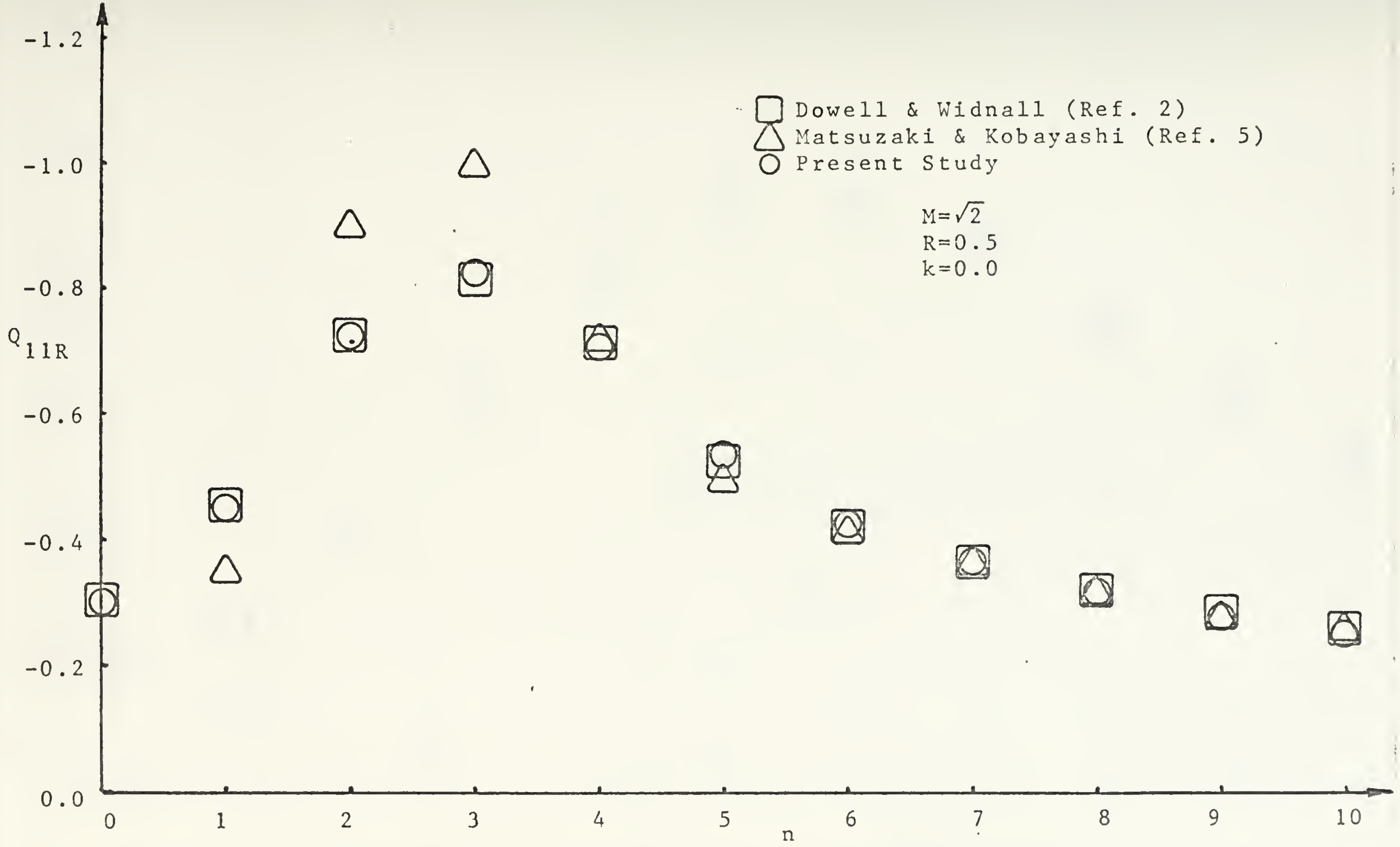


Figure 11. Generalized Aerodynamic Force vs. Circumferential Mode Number

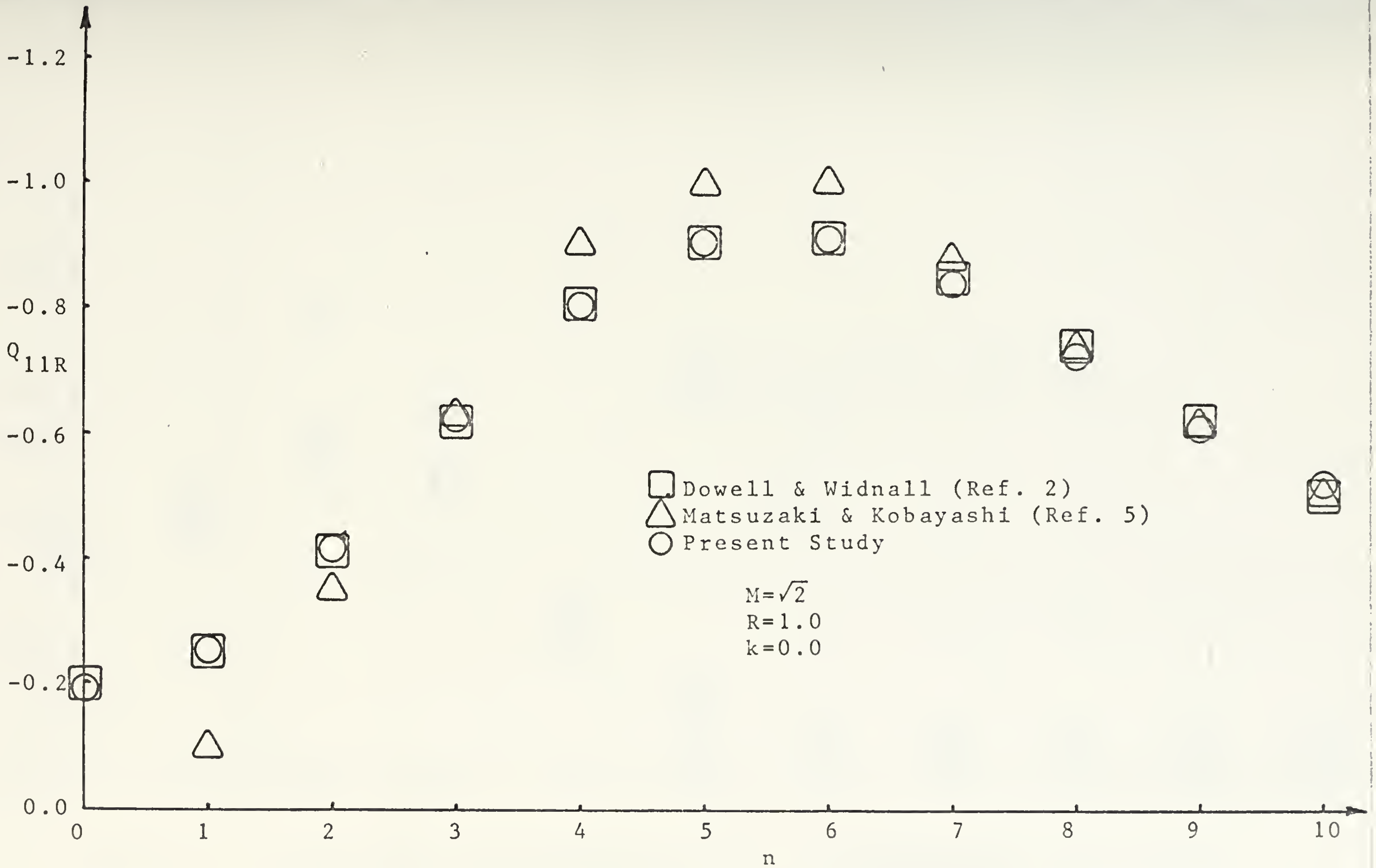


Figure 12. Generalized Aerodynamic Force vs. Circumferential Mode Number

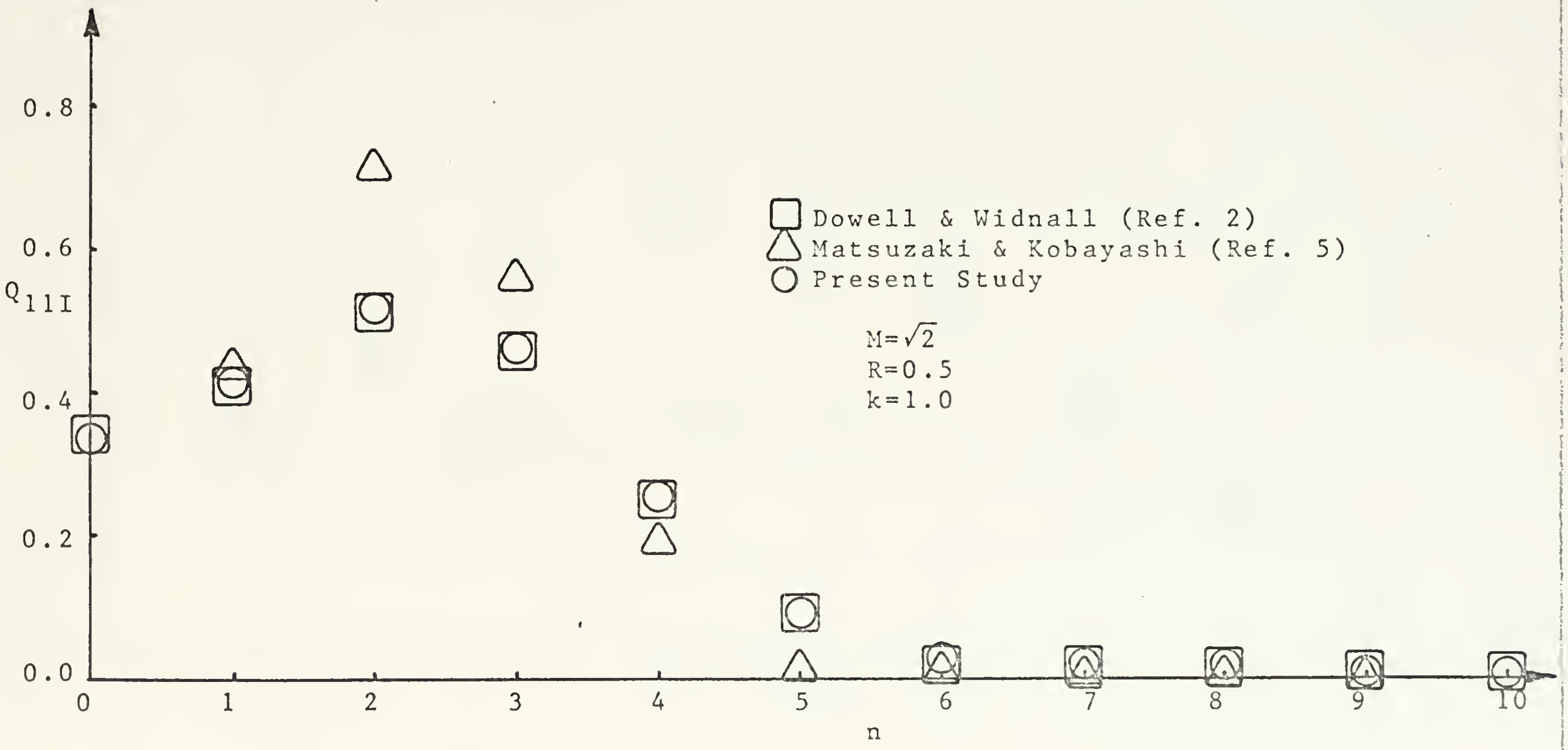


Figure 13. Generalized Aerodynamic Force vs. Circumferential Mode Number

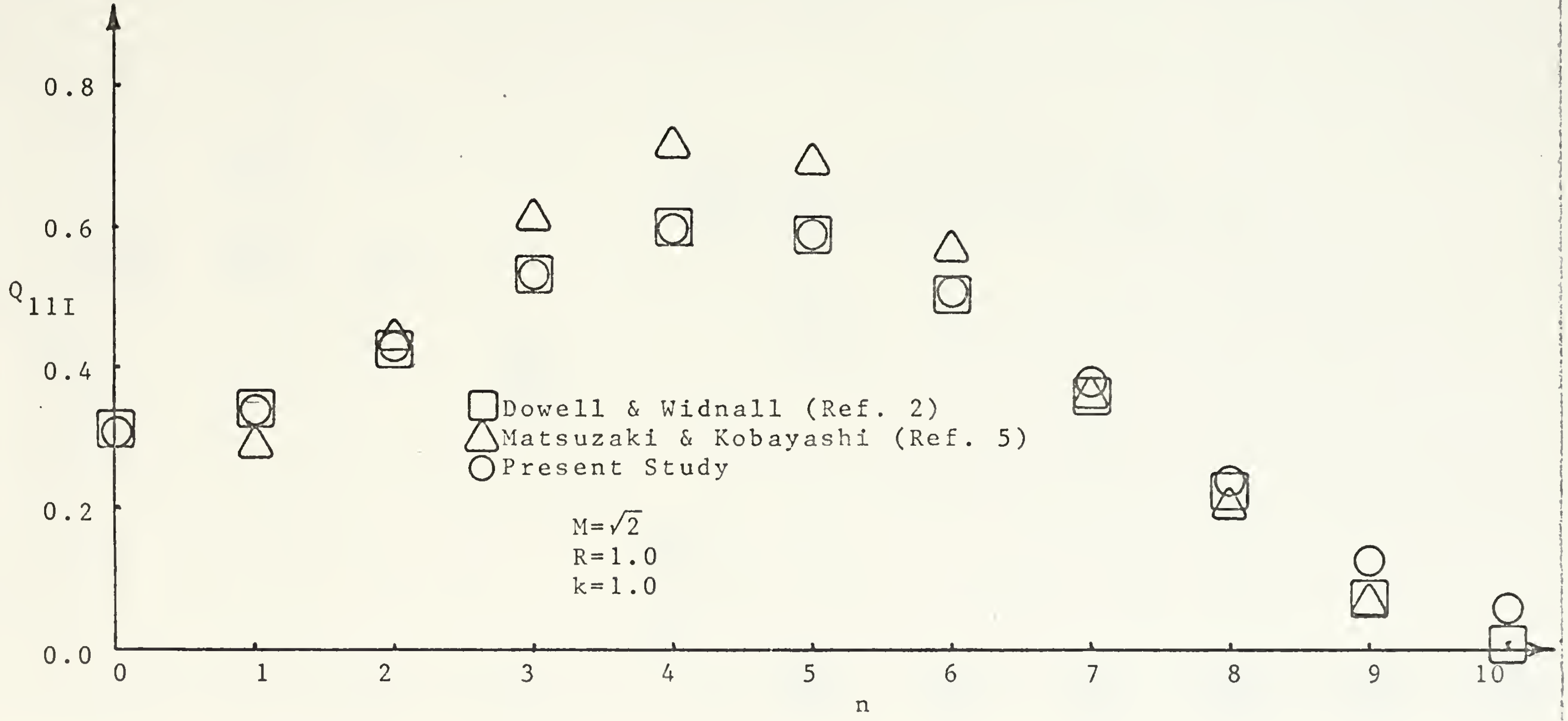


Figure 14. Generalized Aerodynamic Force vs. Circumferential Mode Number

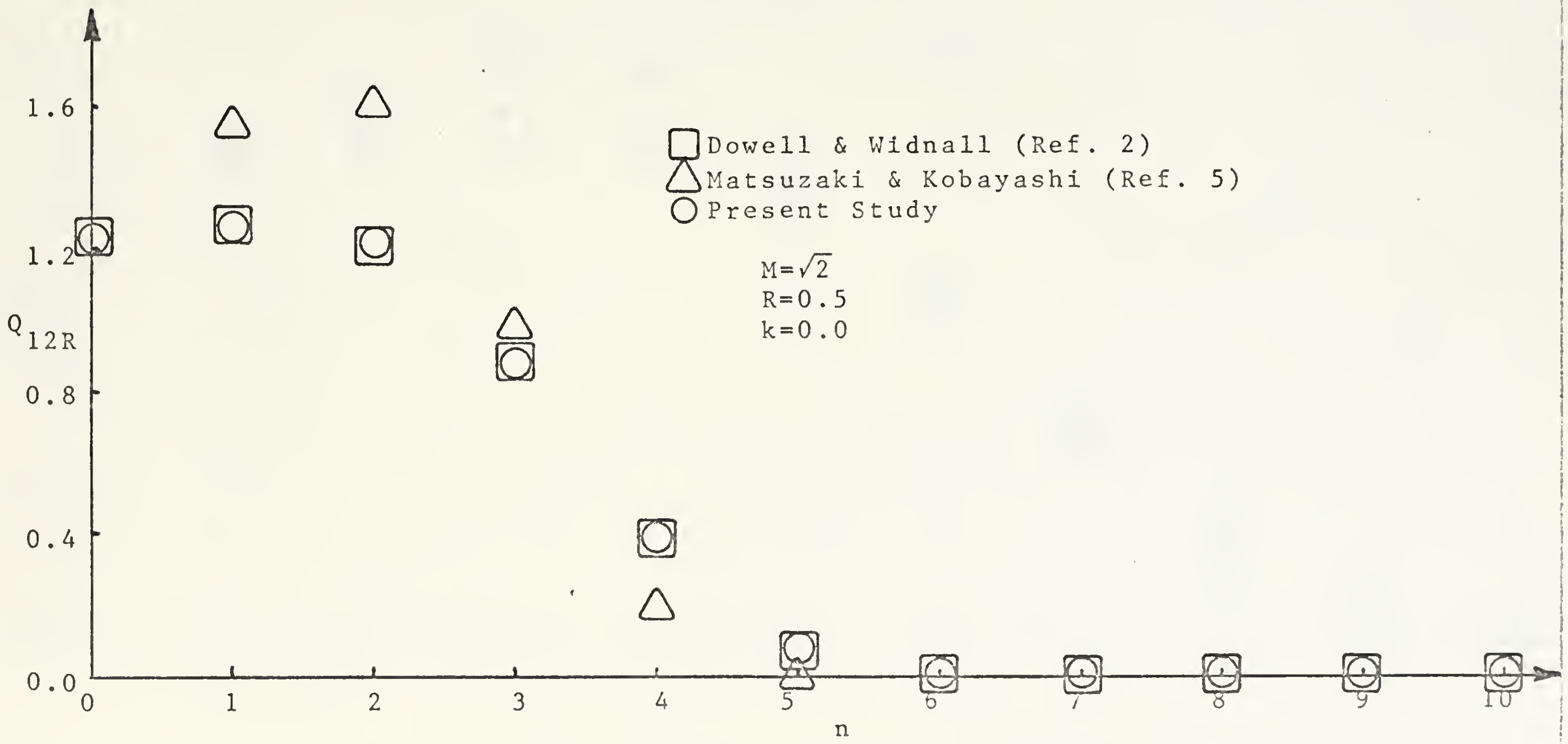


Figure 15. Generalized Aerodynamic Force vs. Circumferential Mode Number

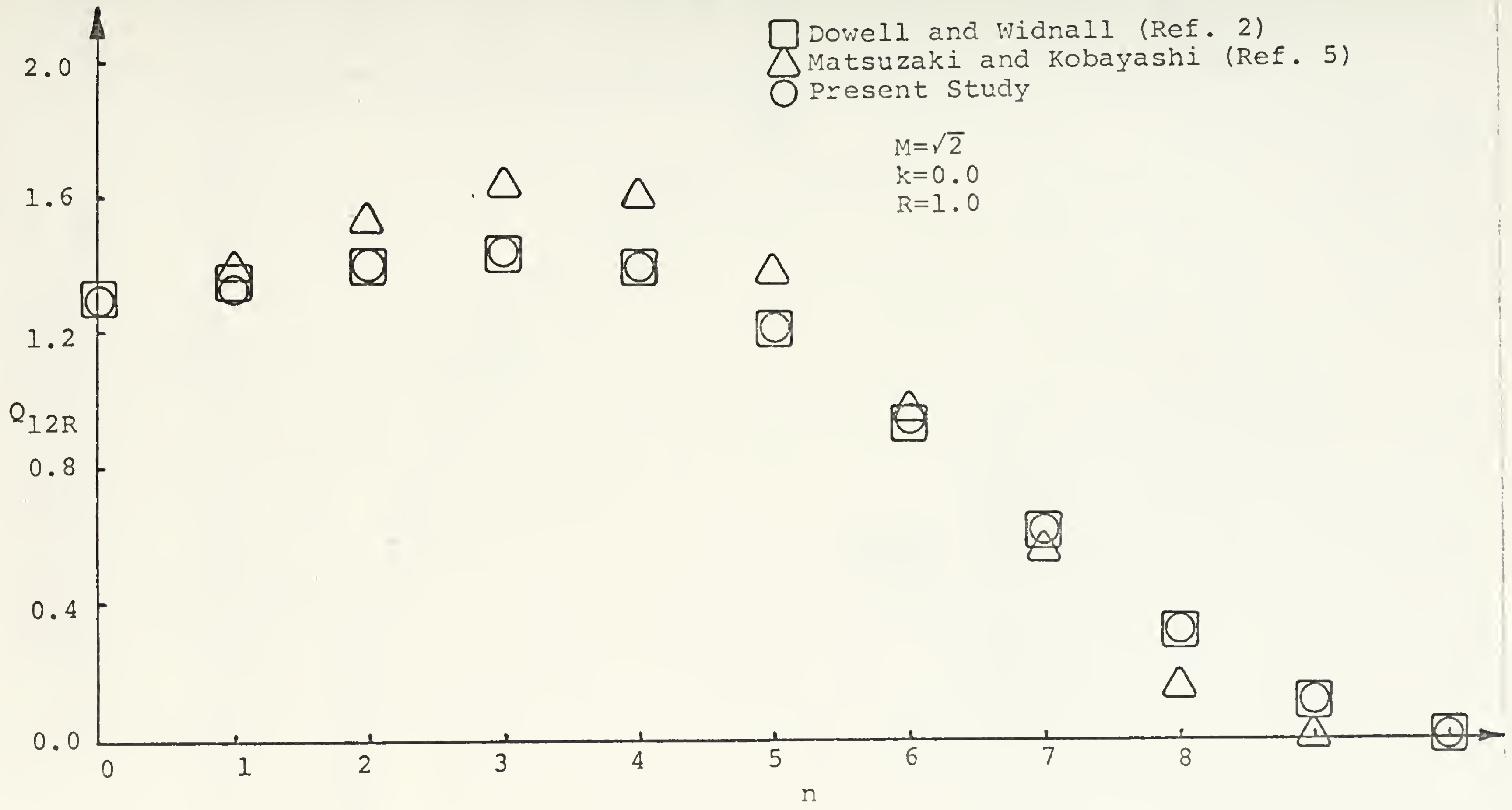


Figure 16. Generalized Aerodynamic Force vs. Circumferential Mode Number

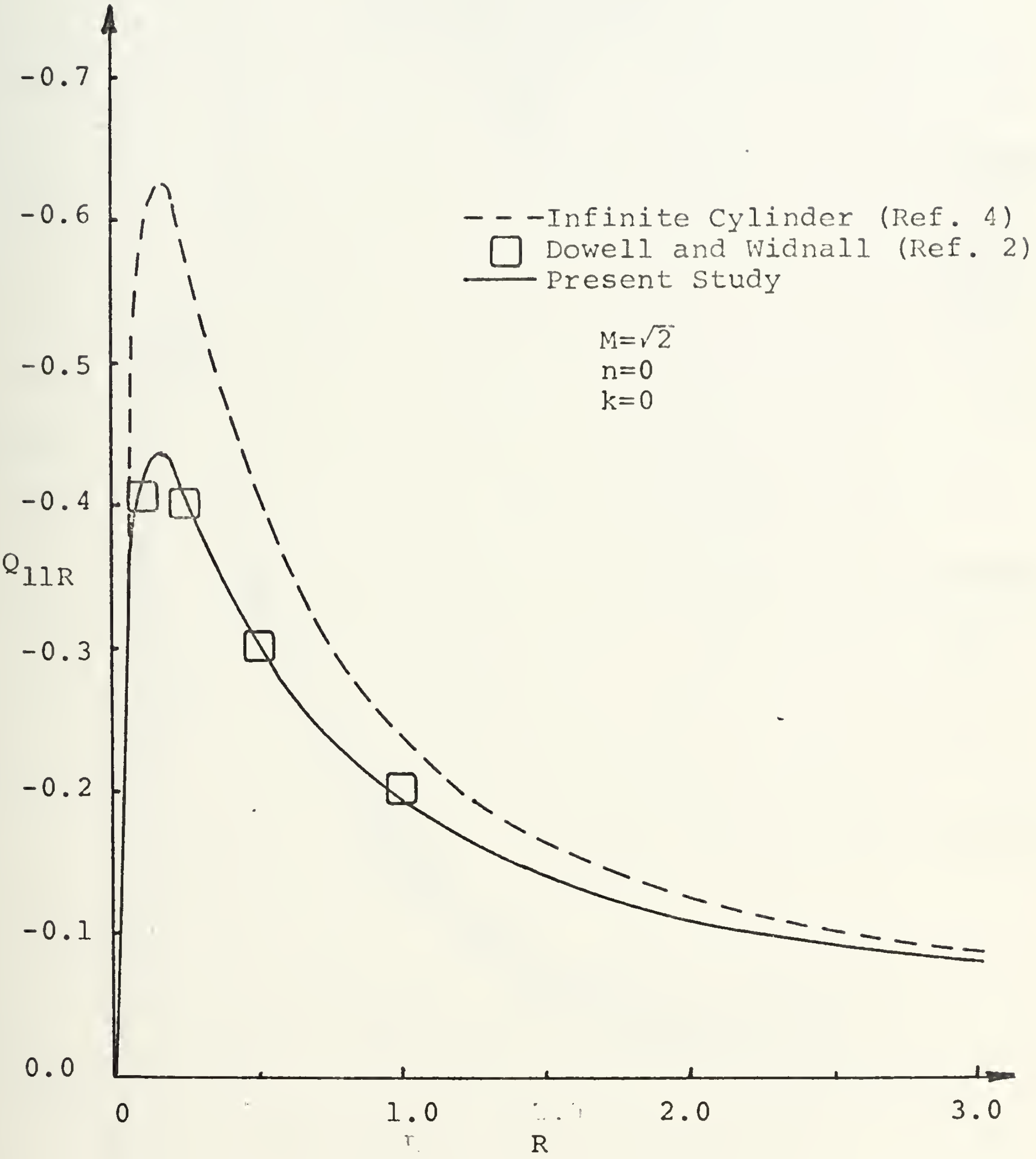


Figure 17. Generalized Aerodynamic Force vs. Radius to Length Ratio

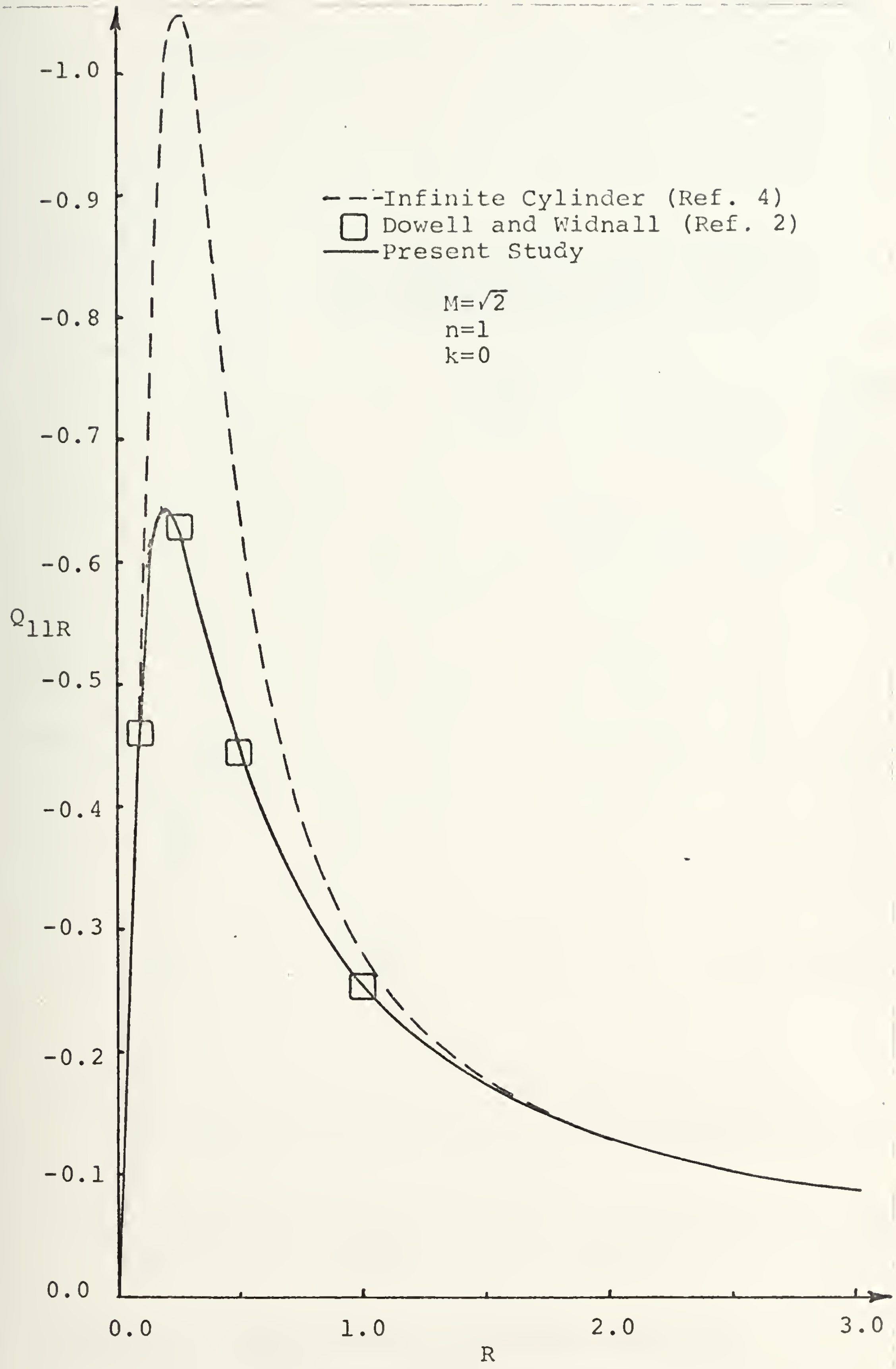


Figure 18. Generalized Aerodynamic Force vs. Radius to Length Ratio

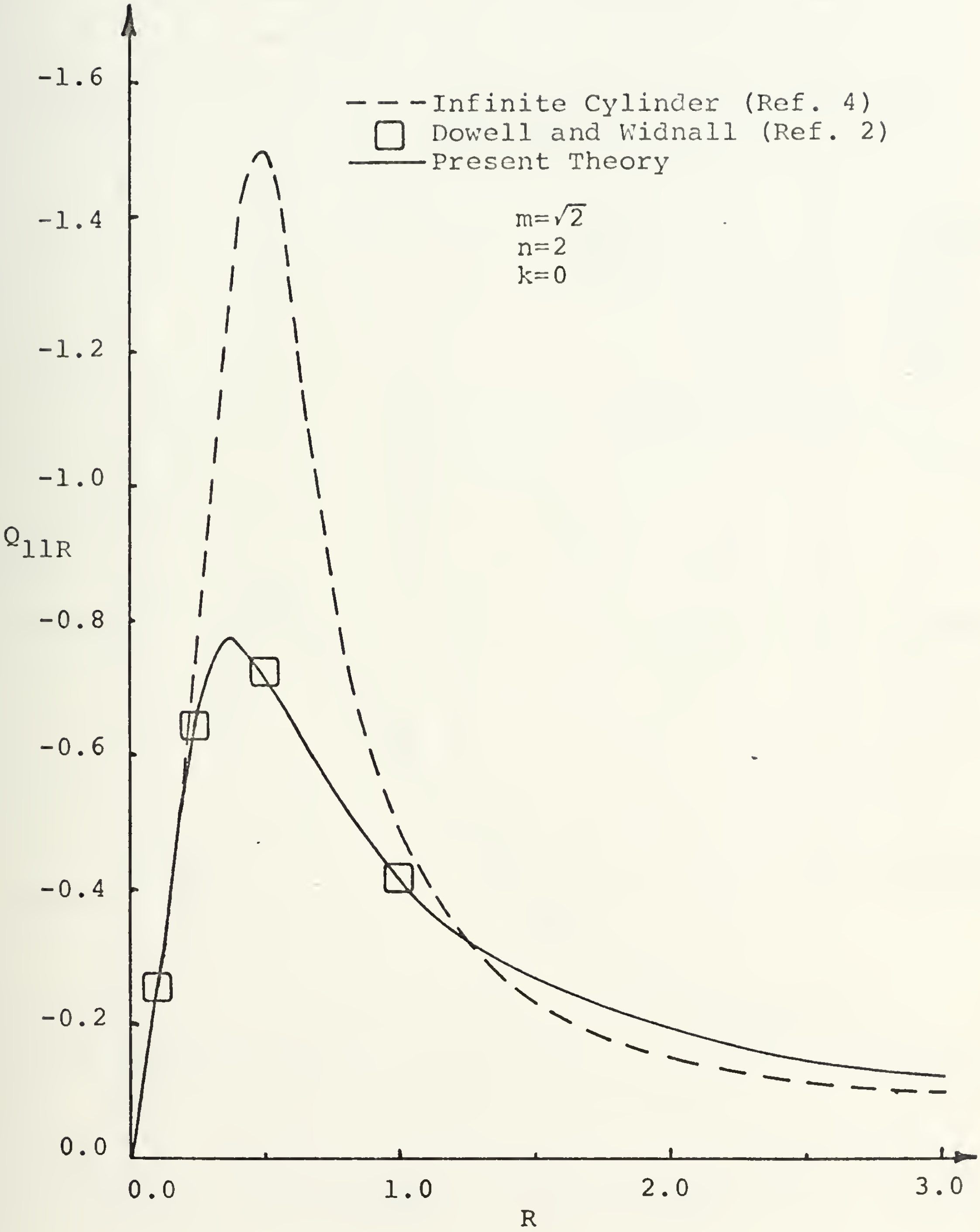


Figure 19. Generalized Aerodynamic Force vs. Radius to Length Ratio

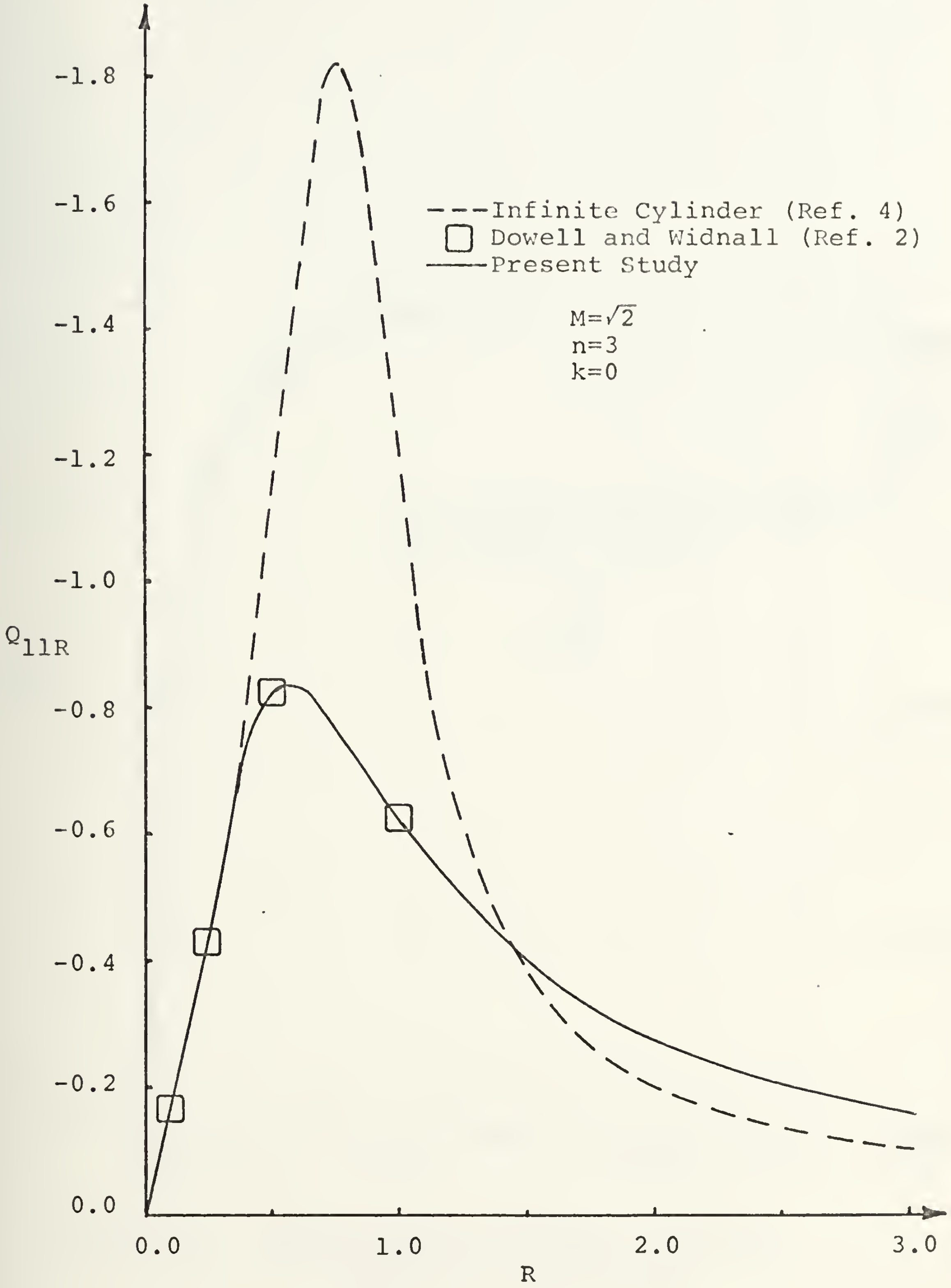


Figure 20. Generalized Aerodynamic Force vs. Radius to Length Ratio

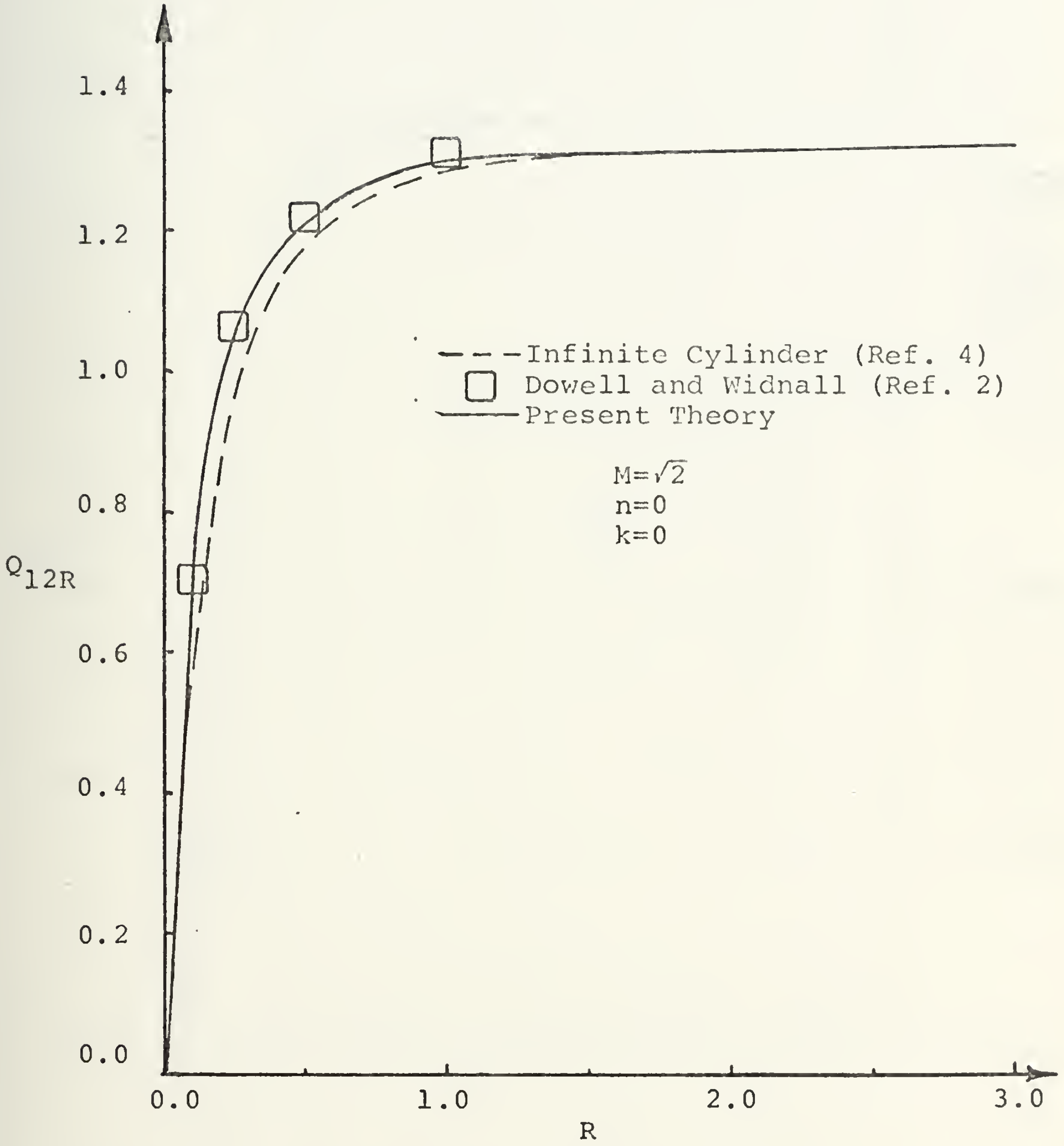


Figure 21. Generalized Aerodynamic Force vs. Radius to Length Ratio

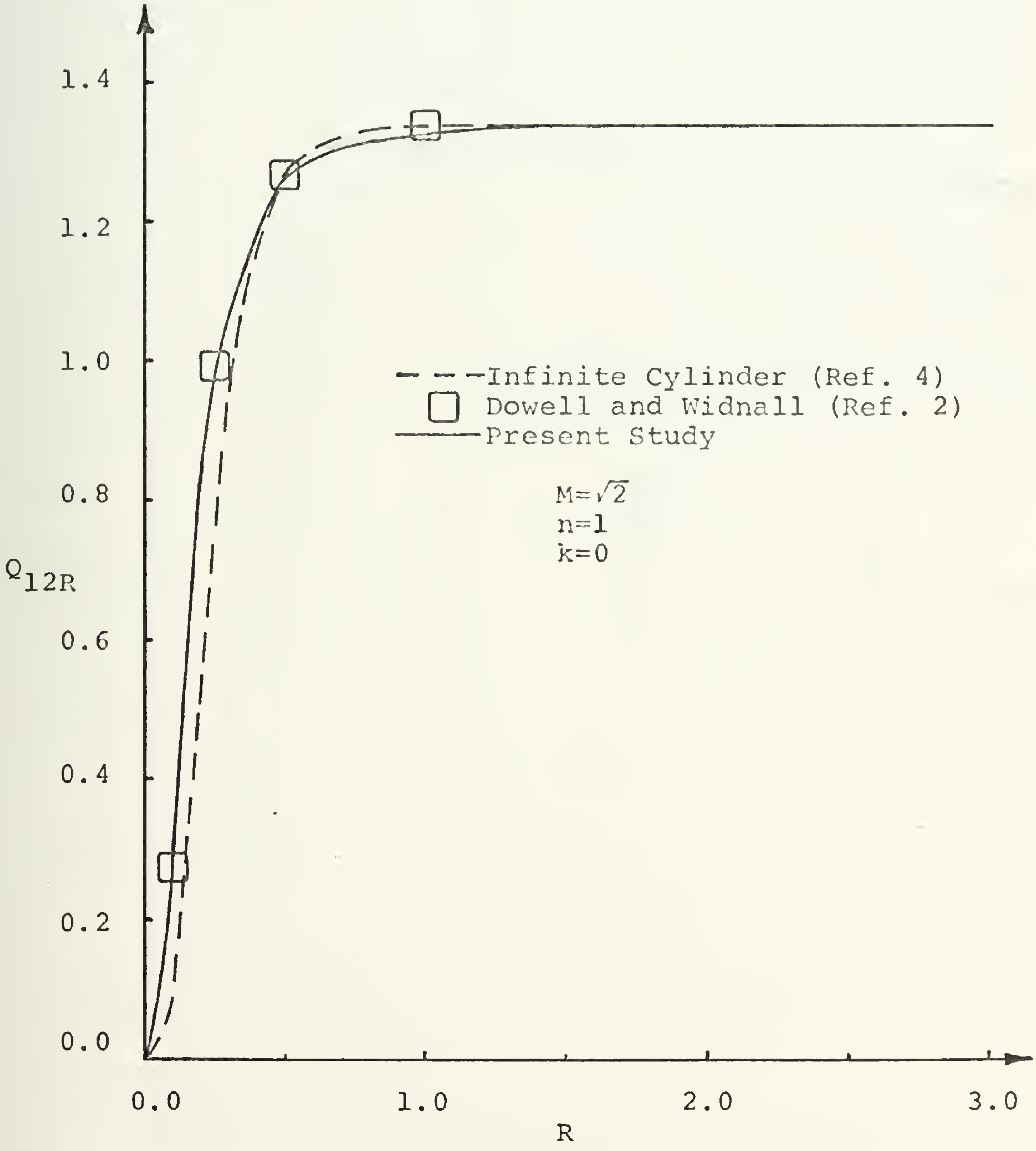


Figure 22. Generalized Aerodynamic Force vs. Radius to Length Ratio

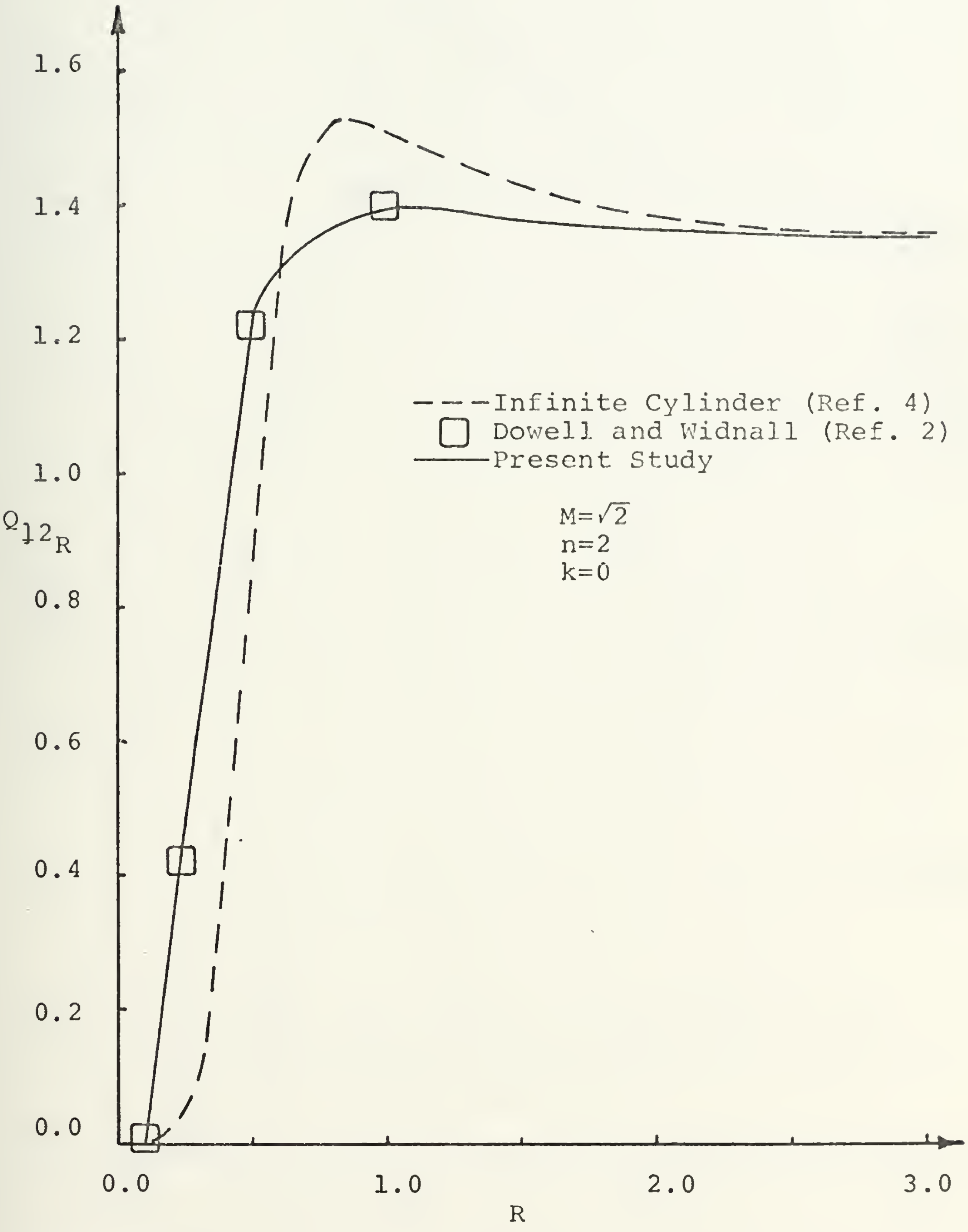


Figure 23. Generalized Aerodynamic Force vs. Radius to Length Ratio

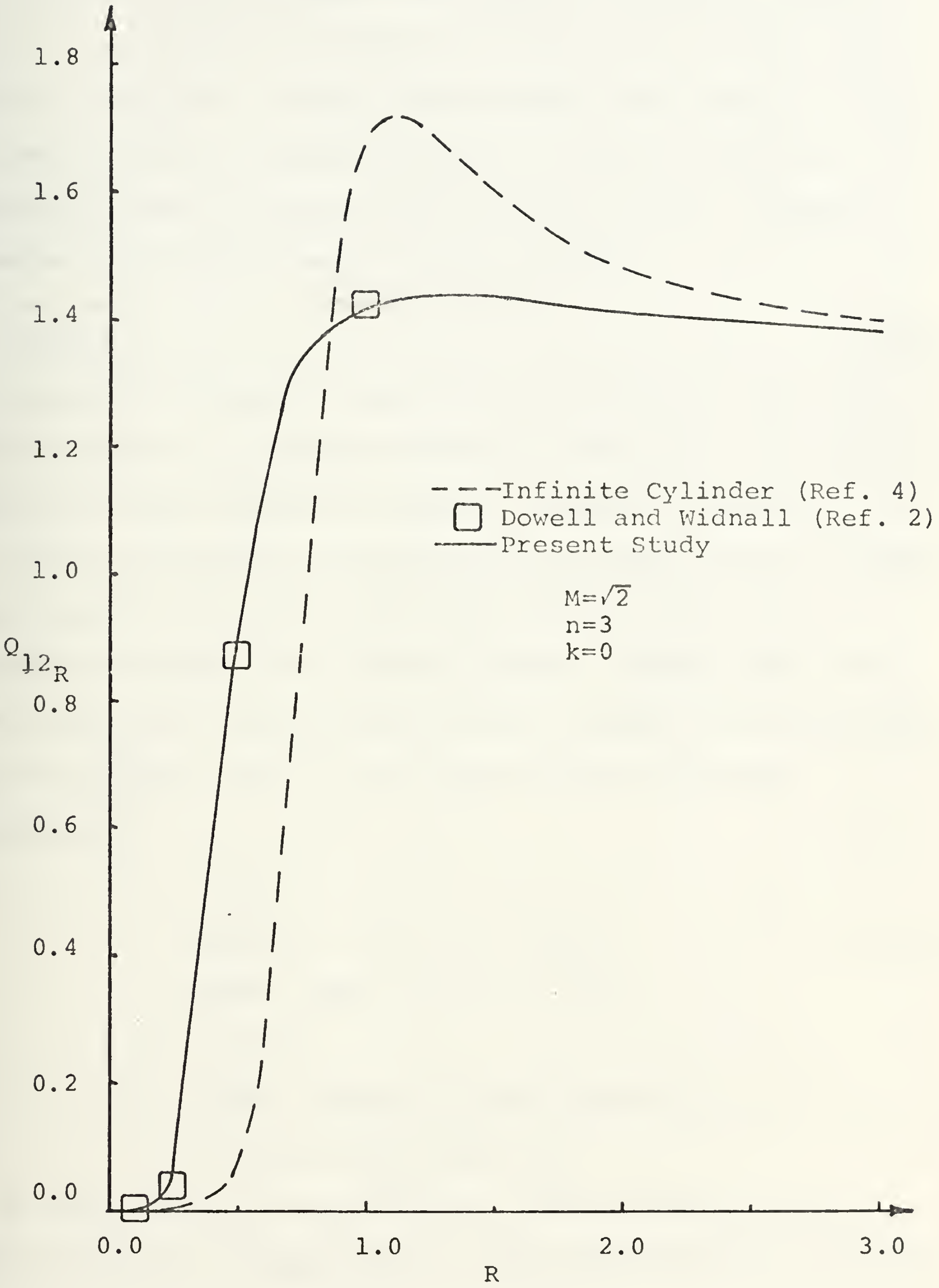


Figure 24. Generalized Aerodynamic Force vs. Radius to Length Ratio

Finally, Figure 25 shows the change in pressure distribution as a function of radius to length ratio for the case $M=\sqrt{2}$, m=1, n=0, k=0. These distributions were computed using the characteristics method outlined in Section III-A and the Sauer-Heinz theory as described in Section III-C. Complete agreement was found between the two methods.

B. CASCADE

Figures 26 through 29 show the instantaneous pressure distributions on the first and second blades of a finite cascade for four different values of reduced frequencies. The pressure difference between lower and upper blade surface is plotted for the instant when the airfoil is at its mean horizontal position and pitching nose up. Results for these cases were given very recently by L. E. Snyder, of Pratt and Whitney Aircraft (Ref. 8), for the following parameter combinations:

$M = 1.5$
Solidity = 1.33 ($d = 0.317$)
Stagger angle = 25° ($\beta = 65°$)
$x_0 = 0.5$
$S = 40°$ (Blade 2 leads blade 1)
$k = 0.04, 0.332, 0.832, 1.25$

It can be seen that appreciable differences exist in a number of cases, in particular in the wave reflection regions. These differences are not fully understood at this time.

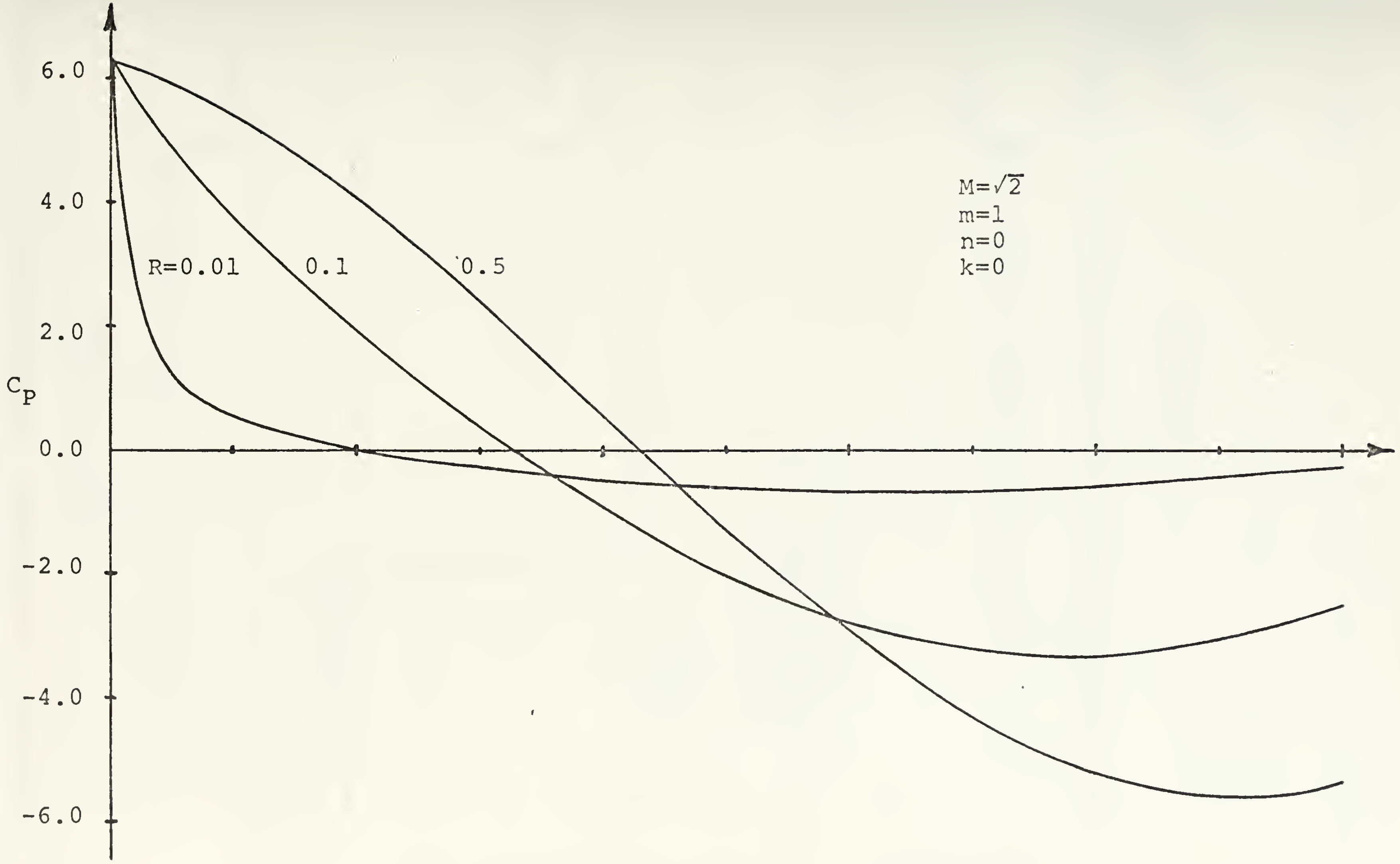


Figure 25. Pressure Distribution Over Wavy-Walled Cylinder

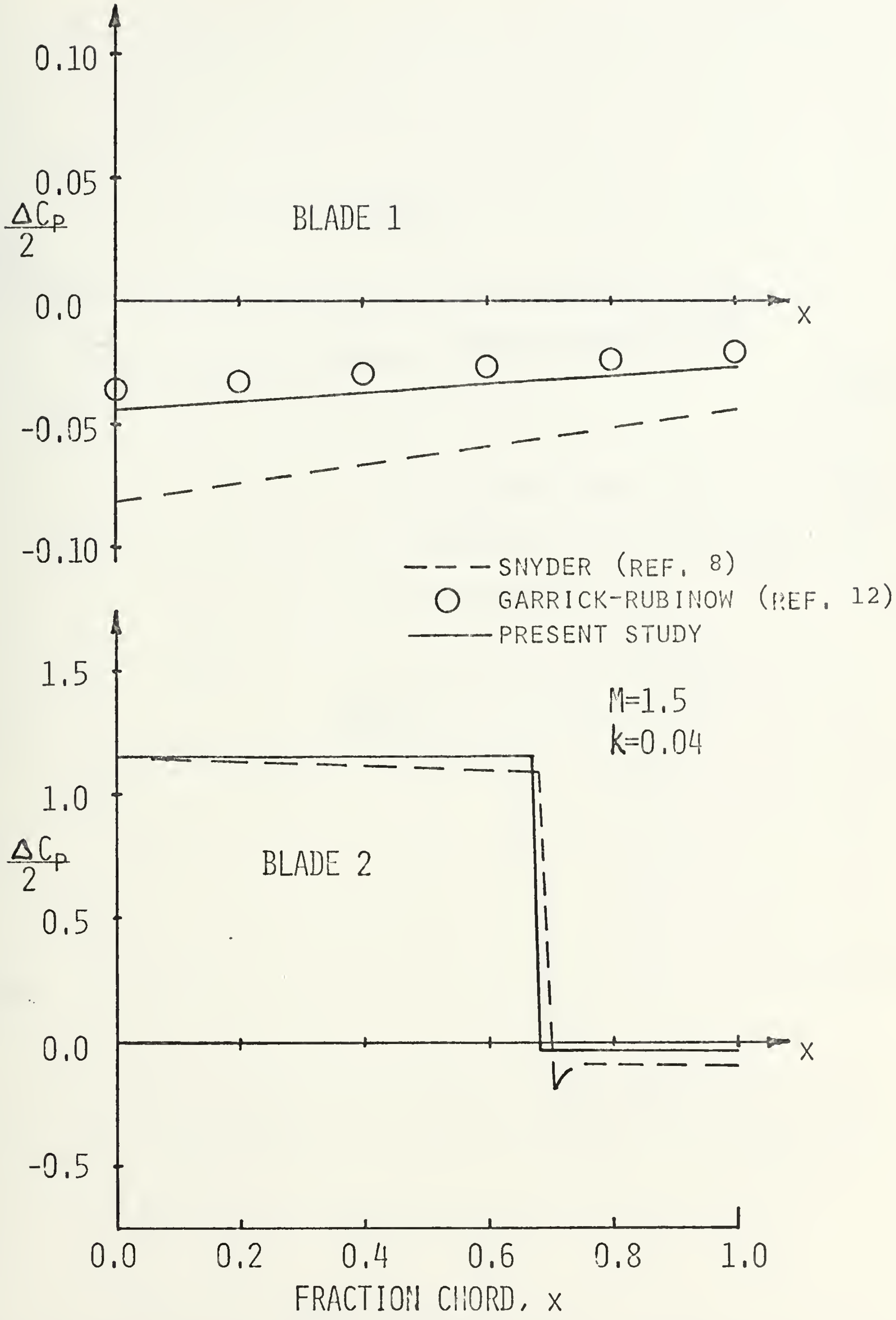


Figure 26. Cascade Pressure Coefficient Difference vs. Fraction Chord

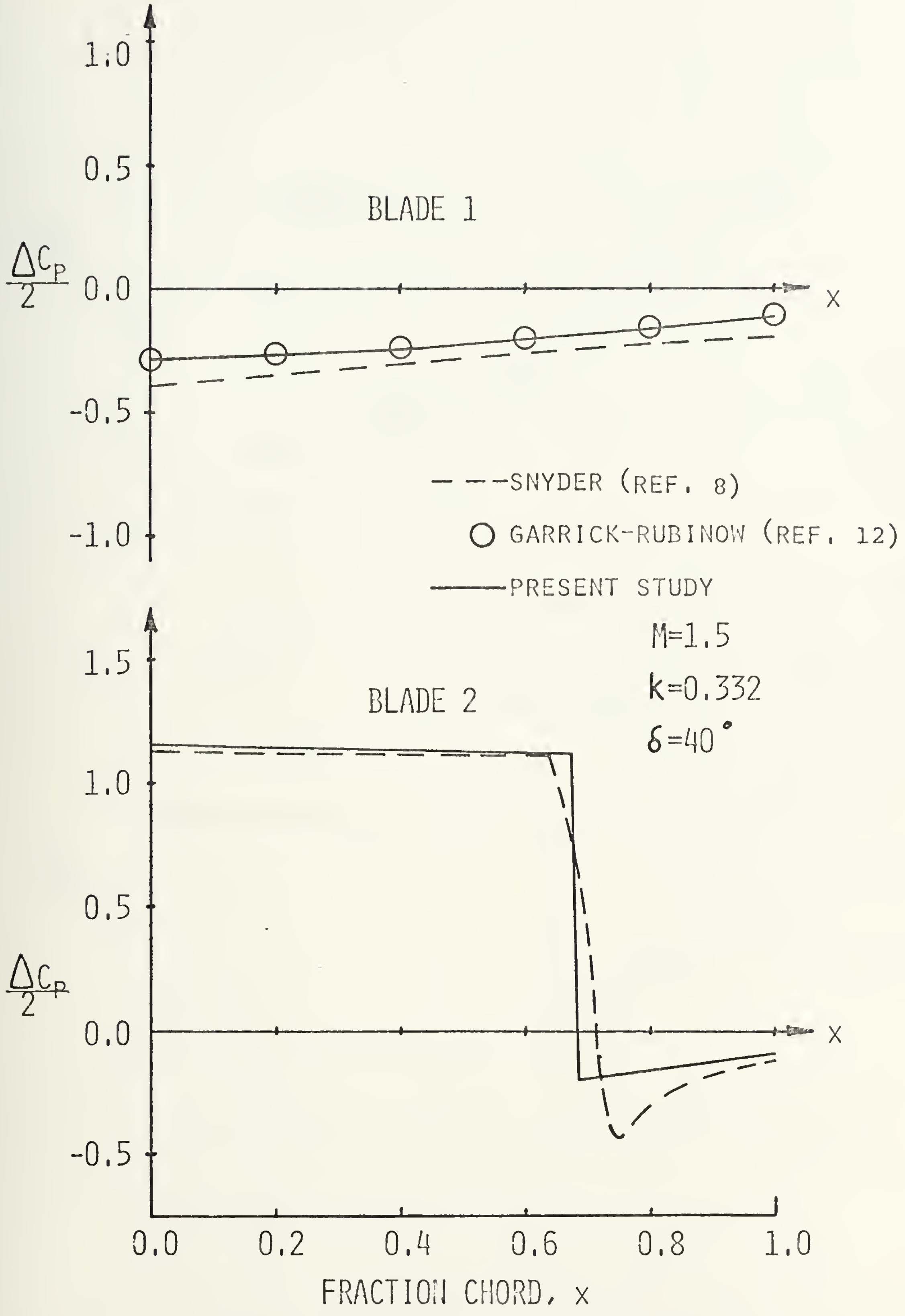


Figure 27. Cascade Pressure Coefficient Difference vs. Fraction Chord

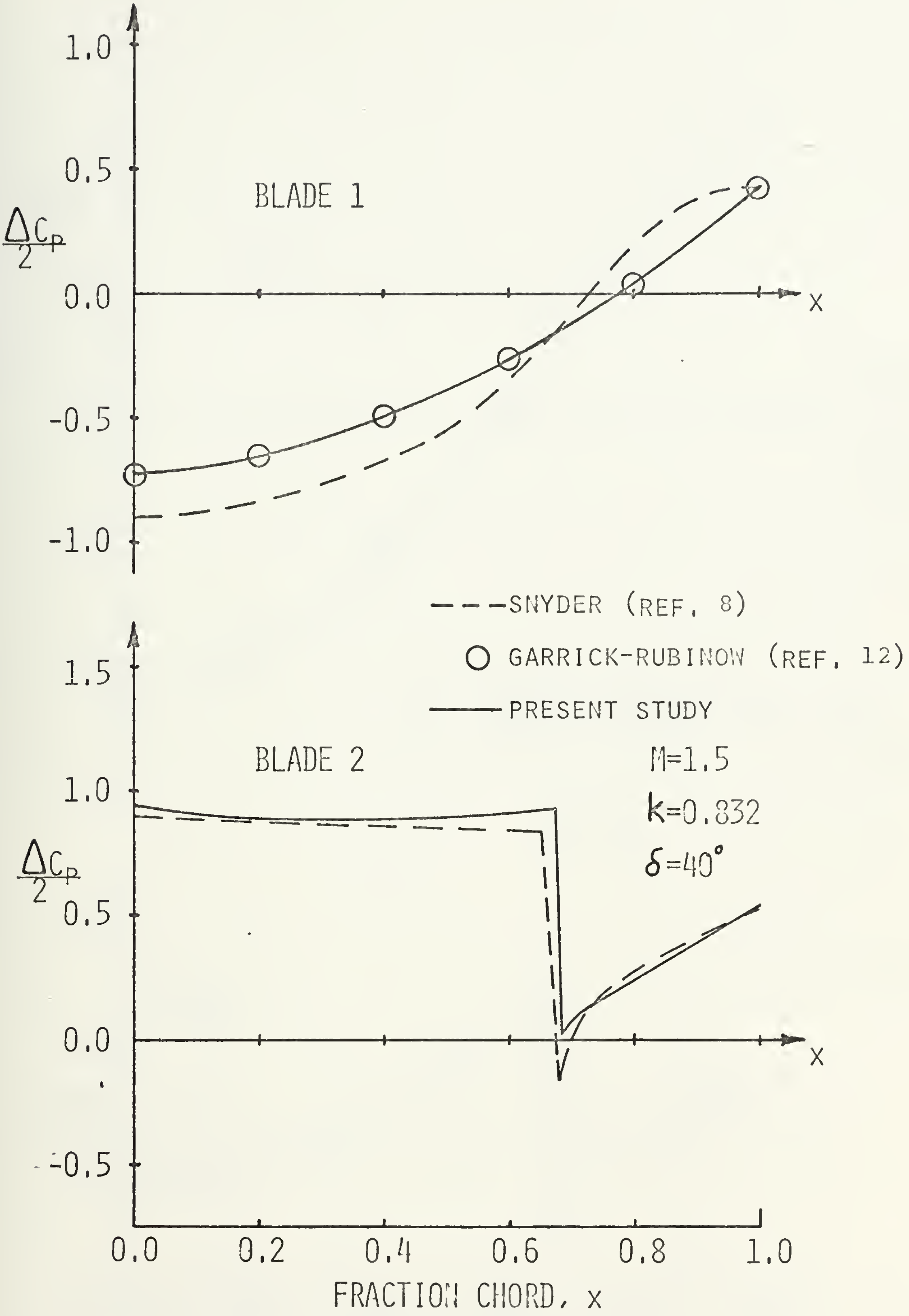


Figure 28. Cascade Pressure Coefficient Difference vs. Fraction Chord

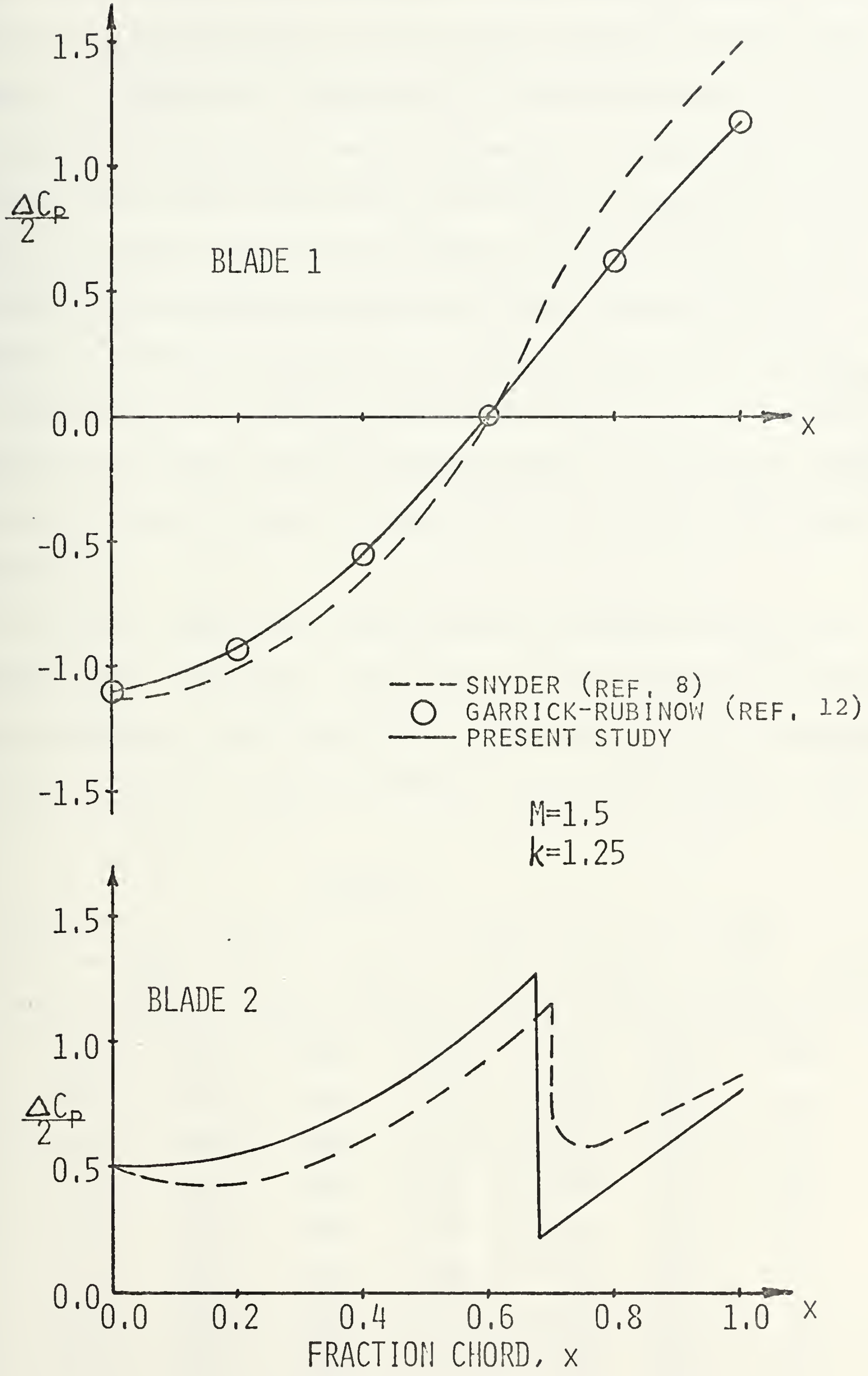


Figure 29. Cascade Pressure Coefficient Difference vs. Fraction Chord

Snyder uses a finite difference procedure and further work is required to evaluate the validity and accuracy of this procedure is compared to the method of characteristics procedure developed in the present work. However, a check could be made for the first blade solutions for which the method of singularities of Garrick and Rubinow (Ref. 12) is available. This solution was programmed according to the equations given in Appendix H and the results are shown in Figures 26 through 29. The method of characteristics solution is in complete agreement with the Garrick-Rubinow prediction whereas considerable deviations are seen to exist for the finite difference predictions.

A further check using the method of singularities was developed for two blades oscillating with zero phase difference. This approach is also described in Appendix H and a comparison with the method of characteristics is given in Table 1 below:

TABLE 1

k	0.1				0.5			
M	$\sqrt{2}$		1.8		$\sqrt{2}$		1.8	
x	M.O.C.	M.O.S.	M.O.C.	M.O.S.	M.O.C.	M.O.S.	M.O.C.	M.O.S.
0.1	.979	.979	.667	.660	.693	.694	.640	.486
0.2	.977	.977	.666	.661	.686	.687	.638	.511
0.3	.974	.974	.666	.662	.680	.681	.635	.536
0.4	.972	.972	.666	.663	.676	.677	.631	.562
0.5	.969	.969	.666	.664	.674	.675	.627	.588
0.6	.966	.966	.665	.666	.675	.676	.623	.614
0.7	.963	.963	.665	.667	.678	.679	.618	.641
0.8	.959	.959	.665	.668	.682	.683	.614	.668
0.9	.956	.956	.664	.669	.689	.690	.607	.694

Further method of characteristics results for a finite subsonic leading edge cascade are shown in Figures 30 through 32 for the following parameter combinations:

$M=\sqrt{2}$ $\quad$ $x_0=0.5$

$d=1.0$ $\quad$ $k=0.5$

$\beta=59.8°$ $\quad$ $\delta=120°$

As mentioned previously, the computer program is written sufficiently general so that the pressure distribution can be computed and printed out for arbitrary specified Mach number, reduced frequency, stagger angle, blade spacing, and number of blades subject to the following constraint:

$$\nu \tan \alpha \left[(N-1)(\tan \beta - \cot \alpha) + \frac{1}{d}\right] < 800 \qquad (227)$$

Equation (227) is the expression which governs the maximum grid fineness that may be used with a given set of input parameters. This constraint is necessary because the program is dimensioned such that the maximum number of grid points allowed on any one right-running Mach line is 400.

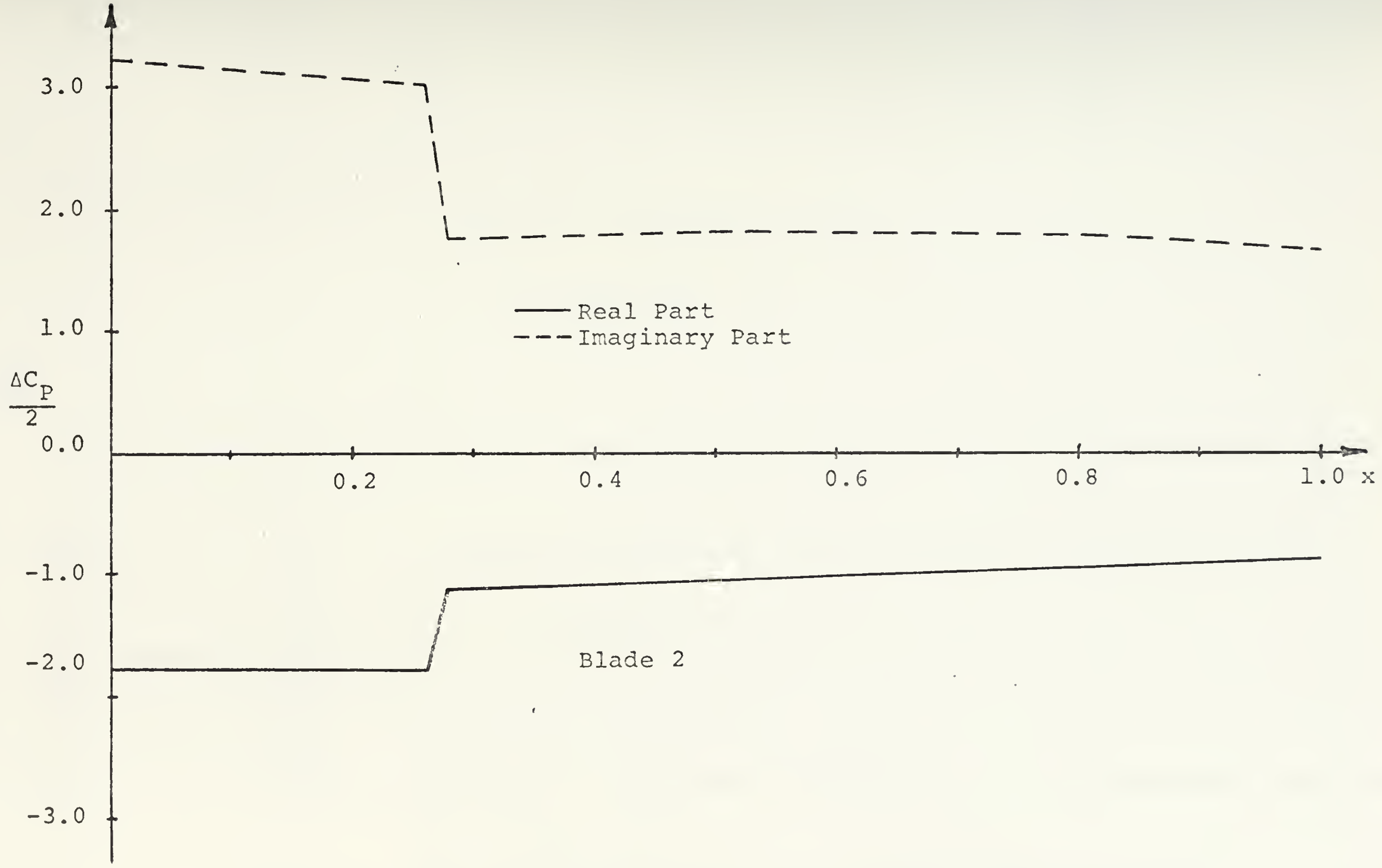


Figure 30. Pressure Coefficient Difference vs. Fraction Chord

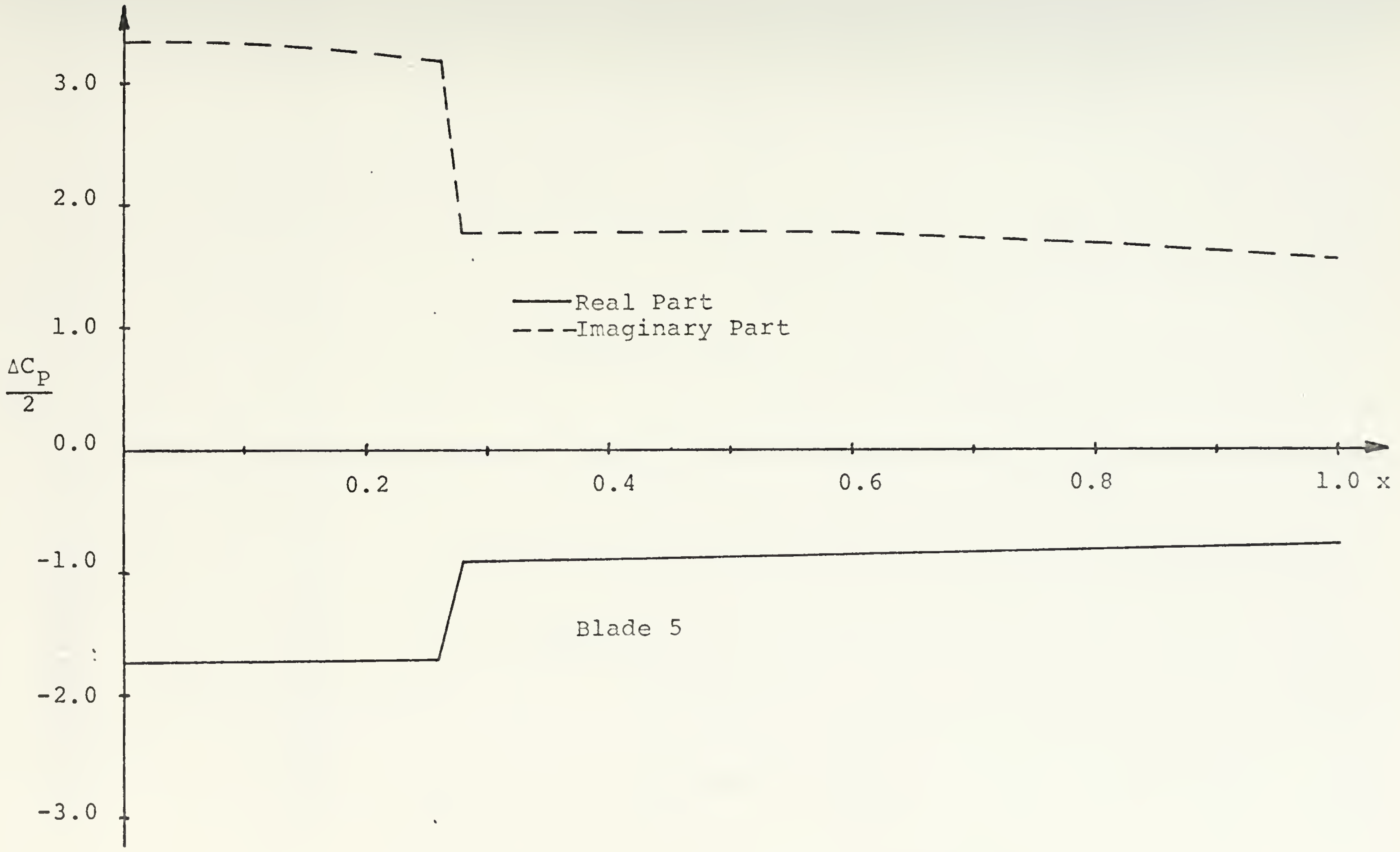


Figure 31. Pressure Coefficient Difference vs. Fraction Chord

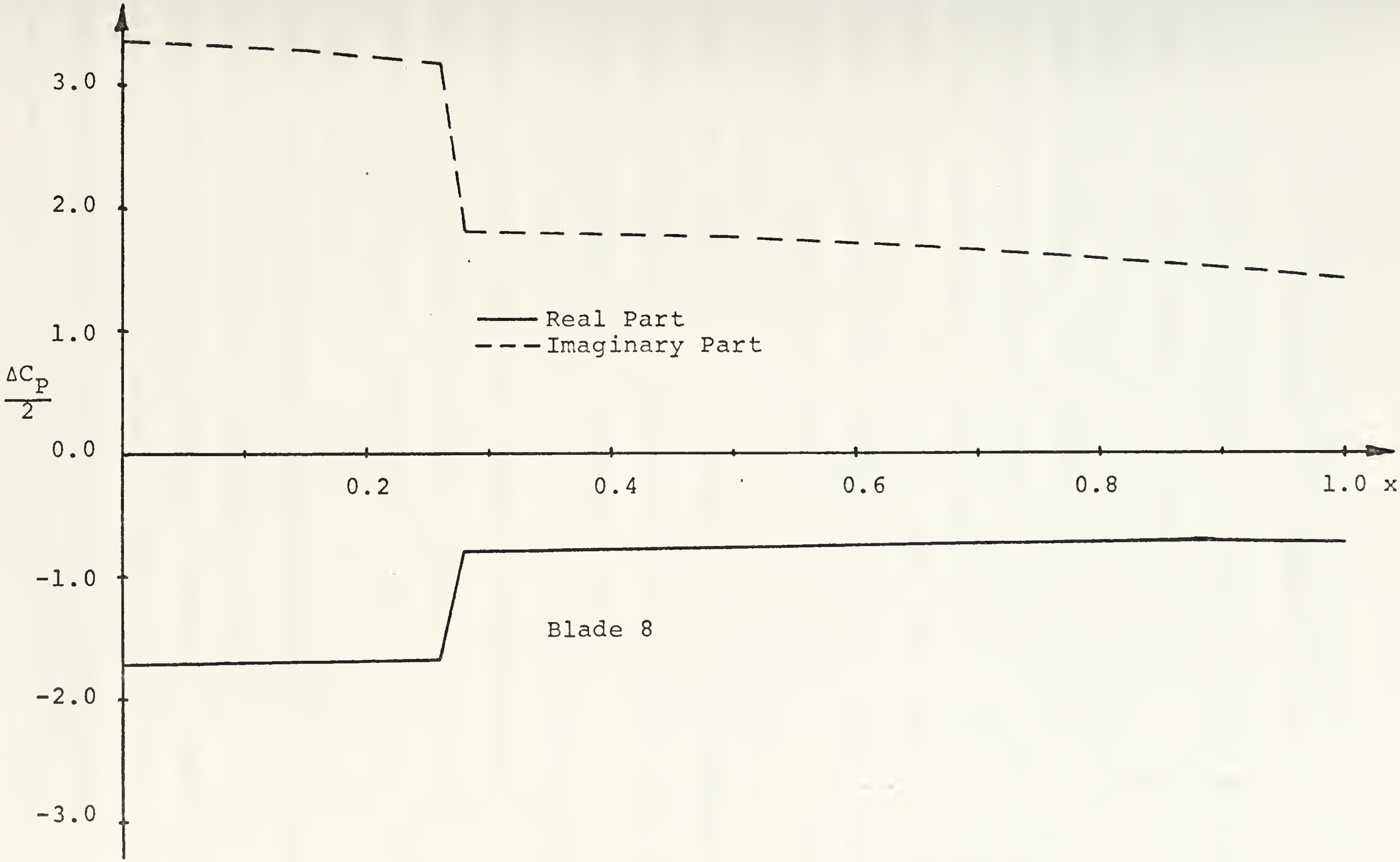


Figure 32. Pressure Coefficient Difference vs. Fraction Chord

VI. CONCLUSIONS AND RECOMMENDATIONS

A solution of the complete linearized unsteady potential equation has been developed for supersonic flow past vibrating cylindrical shells using the method of characteristics. Pressure distributions and generalized aerodynamic forces were computed for shells of arbitrary radius-to-length ratios, axial and circumferential wave numbers, Mach number, and reduced frequencies. A solution for this same problem was previously obtained by Dowell and Widnall using Laplace transform techniques. Excellent agreement was obtained in all cases. It is believed that the present method can be applied more easily than methods requiring complex variable techniques. It should be noted that the procedure can easily be modified to include arbitrary axial deflection modes.

A further solution of the complete linearized unsteady potential equation using the method of characteristics was developed for supersonic flow past vibrating flat plate cascades with subsonic leading edge locus. Pressure distributions were computed for finite cascades of arbitrary stagger and interblade phase angles performing small amplitude pitch oscillations. Preliminary comparisons with a recent finite difference solution by Verdon and Snyder showed reasonable agreement. However, further work is necessary to assess the validity and accuracy of the present method. This should be done by further comparisons with the Verdon and

Snyder procedure and Fourier and Laplace transform methods currently under development at Stevens Institute of Technology, General Motors Corporation (Detroit Diesel, Allison Division), and General Electric. Also, flutter calculations should be carried out using the method of characteristics predictions of the oscillatory aerodynamic forces which would allow comparisons with the Pratt and Whitney cascade flutter tests.

APPENDIX A

CYLINDRICAL SHELL COMPUTER PROGRAM VARIABLES

A. LOGIC VARIABLES

The following variables appear in the logic statements of the program:

IHAVEP	The number of the computation along any right-running Mach line. IHAVEP is used as the first subscript for a grid point.
ISWTCH	Switch variable used to designate the right-running Mach line being calculated, alternately set equal to "1" or "2" and used as a second subscript.
JSWTCH	Switch variable used to designate the previously computed right-running Mach line, always the opposite of ISWTCH and used as a second subscript.
KOUNT	The value of IHAVEP at the cylinder surface.
ML	The number of grid points along the initial Mach line and along the cylinder surface.

B. QUANTITY VARIABLES

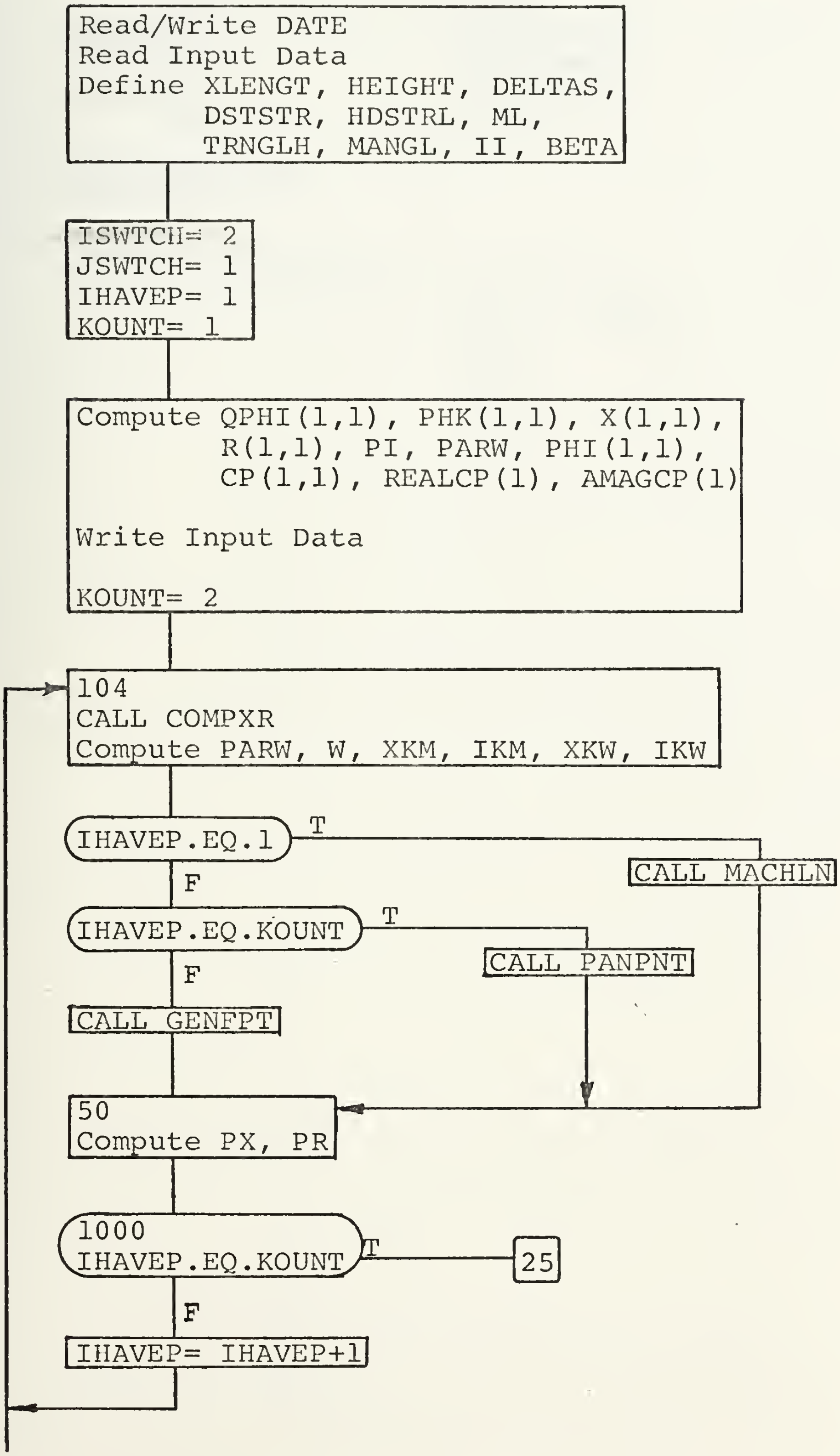
The following variables in the program take on the values defined below. All are floating point quantities unless otherwise stated. The dimensioned variables are listed first.

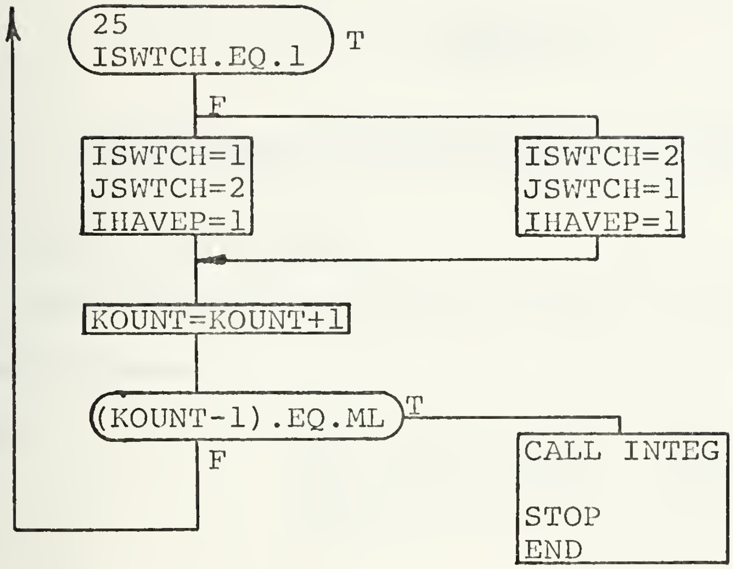
AMAGCP(400)	The imaginary part of the pressure coefficient.
CP(400)	The complex pressure coefficient.
PHI(400,2)	The derivative of the velocity potential along the right-running Mach line.
PHK(400,2)	The derivative of the velocity potential along the left-running Mach line.
QPHI(400,2)	The velocity potential.

REALCP(400,2)	The real part of the pressure coefficient.
X,R(400,2)	The x and r coordinates of a grid point.
A,B,C,D,E,F	See equations (57) - (62)
BETA	The square root of the quantity Mach number squared minus one.
DELTAS	The step size along the characteristics.
DSTSTR	The horizontal distance between grid points.
FINGRD	The grid fineness ratio. An even integer less than 400.
HDSTRL	One-half DSTSTR.
HEIGHT	The increment HDSTRL $\frac{1}{2}$ tan α.
II	The square root of minus one.
IKM	The factor, ik $\frac{M}{M^2-1}$
IKW	The quantity, ik sin $m\pi x$
IPHI1, IPHI2	The velocity potential as described in equations (51) and (54).
K	The reduced frequency.
MACH	The free-stream Mach number
MANGL	The Mach angle, α.
MFREQ	The axial mode number. An integer.
NFREQ	The circumferential mode number. An integer.
NR	r, as defined in equation (68). An integer.
PARW	The partial derivative of the deflection taken with respect to x.
PI	3.141593
PX	The partial derivative of the velocity potential taken with respect to x.
RADIUS	The radius to length ratio.
W	The cylinder deflection.

APPENDIX B

CYLINDRICAL SHELL FLOW DIAGRAM




25
ISWTCH.EQ.1
T
F
ISWTCH=1
JSWTCH=2
IHAVEP=1
ISWTCH=2
JSWTCH=1
IHAVEP=1
KOUNT=KOUNT+1
(KOUNT-1).EQ.ML
T
F
CALL INTEG
STOP
END

APPENDIX C

OSCILLATING CASCADE COMPUTER PROGRAM VARIABLES

A. LOGICAL VARIABLES

The following variables appear in the logic statements of the program:

ICO	Set equal to "1" when a blade or wake computation is made. Set equal to "0" after the next general flow field computation occurs.
IHAVEP	The number of the computation along any right-running Mach line starting with zero at the initial left-running Mach line. The value IHAVEP+1 is the first subscript for a flow quantity.
ISWTCH	Switch variable used to designate the right-running Mach line being calculated, alternately set equal to "1" or "2" and used as a second subscript.
JLINE	The Mach line counter incremented each time a right-running Mach line is completed. Reset equal to the value of IHAVEP at the leading edge of a new blade.
JSWTCH	Switch variable used to designate the previously computed right-running Mach line, always the opposite of ISWTCH and used as a second subscript.
LCOUNT	The value of IHAVEP at a surface or wake grid point.
LIMIT	The maximum value of IHAVEP.
LIMW	The value of LCOUNT at the trailing edge of a blade.
MAXI	The value of JLINE on a right-running Mach line which intersects a blade's leading edge.
NEWDST	The number of Δx increments in a blade chord.

NSTPTS	The number of Δx increments between the initial Mach line and the leading edge of the first blade.
NUM	Set equal to one less than the number of the blade encountered on a Mach line.
NUMBLD	The number of blades in the cascade.
NUMOLD	Preserves the maximum value of NUM.
NUMPI	NUMOLD plus one.
OLDL	Preserves the value of LCOUNT at the highest blade or wake yet encountered.

B. QUANTITY VARIABLES

The following variables in the program take on the values defined below. All are floating point quantities unless otherwise stated. The dimensioned variables are listed first.

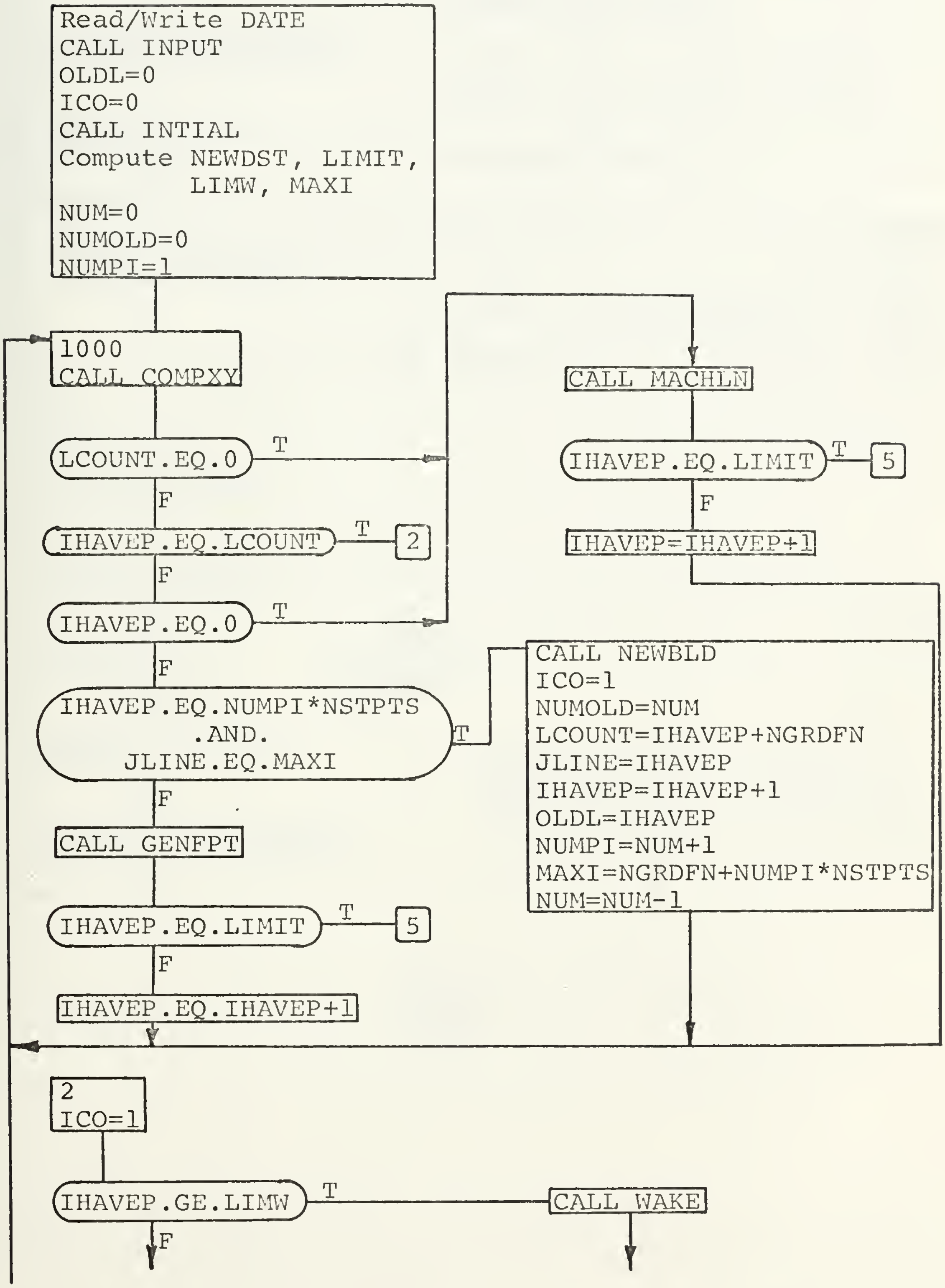
A(8,8)	Matrix of coefficients in wake computation.
B(8)	Right-hand side of equations (194) - (201)
U22R,U22I,V22R,V22I,C22R,C22I(400,2).	The velocities, real and imaginary of general flow field, upper blade, and upper wake points. Equivalent to the respective Teipel amplitude functions, U_{22R} through C_{22I}.
U33R,U33I,V33R,V33I,C33R,C33I(400,2).	Same as above, but used only at lower blade and lower wake points.
X,Y(400,2)	The x and y coordinates based on chord length.
AI	A_I as defined in equation (122).
ARG	The argument $k \dfrac{M^2}{M^2-1} x$
BI	B_I as defined in equation (123).
DELTA	The interblade phase angle in radians.
DELTAS	Incremental distance between grid points along Mach line.
DSTSTR	The increment Δx.

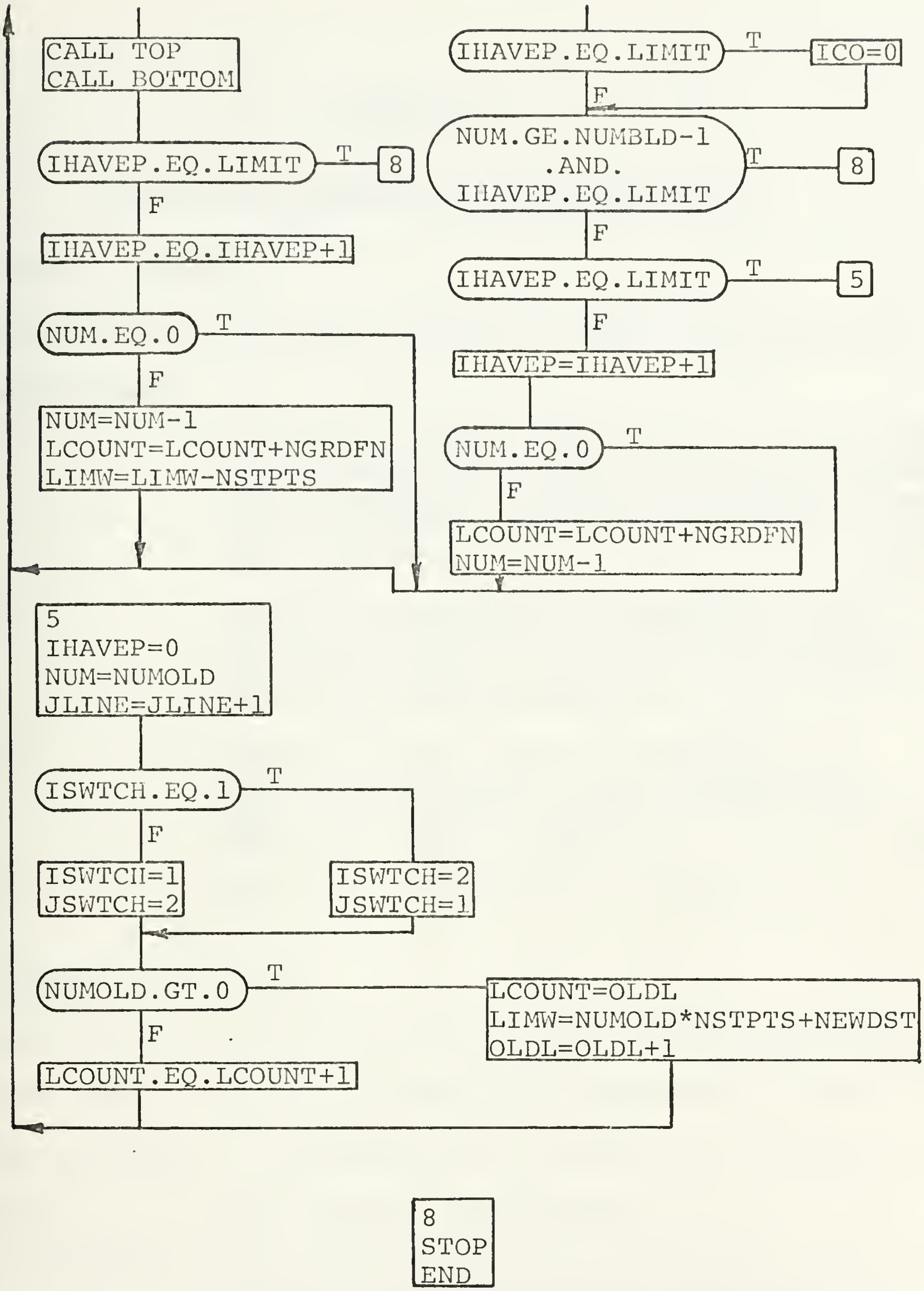
FAZE	The interblade phase angle in degrees.
FMANGL	Mach angle, α.
FNGDPT	Floating point NGRDFN.
FSTRMN	The free-stream Mach number.
G1,...,G9	Intermediate factors used in velocity computations in GENFPT, TOP, BOTTOM, and NEWBLD.
HDSTRL	The increment $\Delta x/2$.
K12R,...,K56I	$K_{12R},\ldots,K_{56I}$ as defined in equations (116) - (121).
KS	Dummy variable used in SIMQ. An integer.
NGRDFN	The grid fineness ratio. An even integer, less than 400.
REDFRQ	The reduced frequency.
STAG	The distance the second blade is staggered back from the y-axis.
STGR	The distance from the initial Mach line to the second blade's leading edge.
STGANG	The stagger angle in degrees.
S	The factor $\sqrt{M^2-1}$.
TNWDST	The vertical distance between adjacent blades.
TRNGLH	The increment $\frac{1}{2}\Delta x \cos \alpha$.
U	The factor $\cos(k \frac{M^2}{M^2-1} x)$.
V	The factor $\sin(k \frac{M^2}{M^2-1} x)$.
VIPANL	The imaginary part of the normal flow velocity at the first blade's leading edge.
VRPANL	The real part of the normal flow velocity at the first blade's leading edge.
W	The factor $\frac{M^2}{\sqrt{M^2-1}}$

XSUBO	The elastic axis position based on chord length.
XLNGTH	The length of the initial Mach line between the planes of adjacent blades.
XN	Floating point NUM.
XNEW	The x-coordinate on a blade based on the blade's leading edge.

APPENDIX D

CASCADE FLOW DIAGRAM




CALL TOP
CALL BOTTOM
IHAVEP.EQ.LIMIT
T
8
F
IHAVEP.EQ.IHAVEP+1
NUM.EQ.0
T
F
NUM=NUM-1
LCOUNT=LCOUNT+NGRDFN
LIMW=LIMW-NSTPTS
IHAVEP.EQ.LIMIT
T
ICO=0
F
NUM.GE.NUMBLD-1
.AND.
IHAVEP.EQ.LIMIT
T
8
F
IHAVEP.EQ.LIMIT
T
5
F
IHAVEP=IHAVEP+1
NUM.EQ.0
T
F
LCOUNT=LCOUNT+NGRDFN
NUM=NUM-1
5
IHAVEP=0
NUM=NUMOLD
JLINE=JLINE+1
ISWTCH.EQ.1
T
F
ISWTCH=1
JSWTCH=2
ISWTCH=2
JSWTCH=1
NUMOLD.GT.0
T
F
LCOUNT=OLDL
LIMW=NUMOLD*NSTPTS+NEWDST
OLDL=OLDL+1
LCOUNT.EQ.LCOUNT+1
8
STOP
END

APPENDIX E

SAUER-HEINZ COMPUTER PROGRAM VARIABLES

A. LOGIC VARIABLES

The following variables appear in the logic statements of the program:

IHAVEP	The number of the computation along any right-running Mach line starting with zero at the initial left-running Mach line. The value IHAVEP+1 is the first subscript for a flow quantity.
ISWTCH	Switch variable used to designate the right-running Mach line being calculated, alternately set equal to "1" or "2" and used as a second subscript.
JSWTCH	Switch variable used to designate the previously computed right-running Mach line, always the opposite of ISWTCH and used as a second subscript.
KOUNT	The value of IHAVEP at the surface.

B. QUANTITY VARIABLES

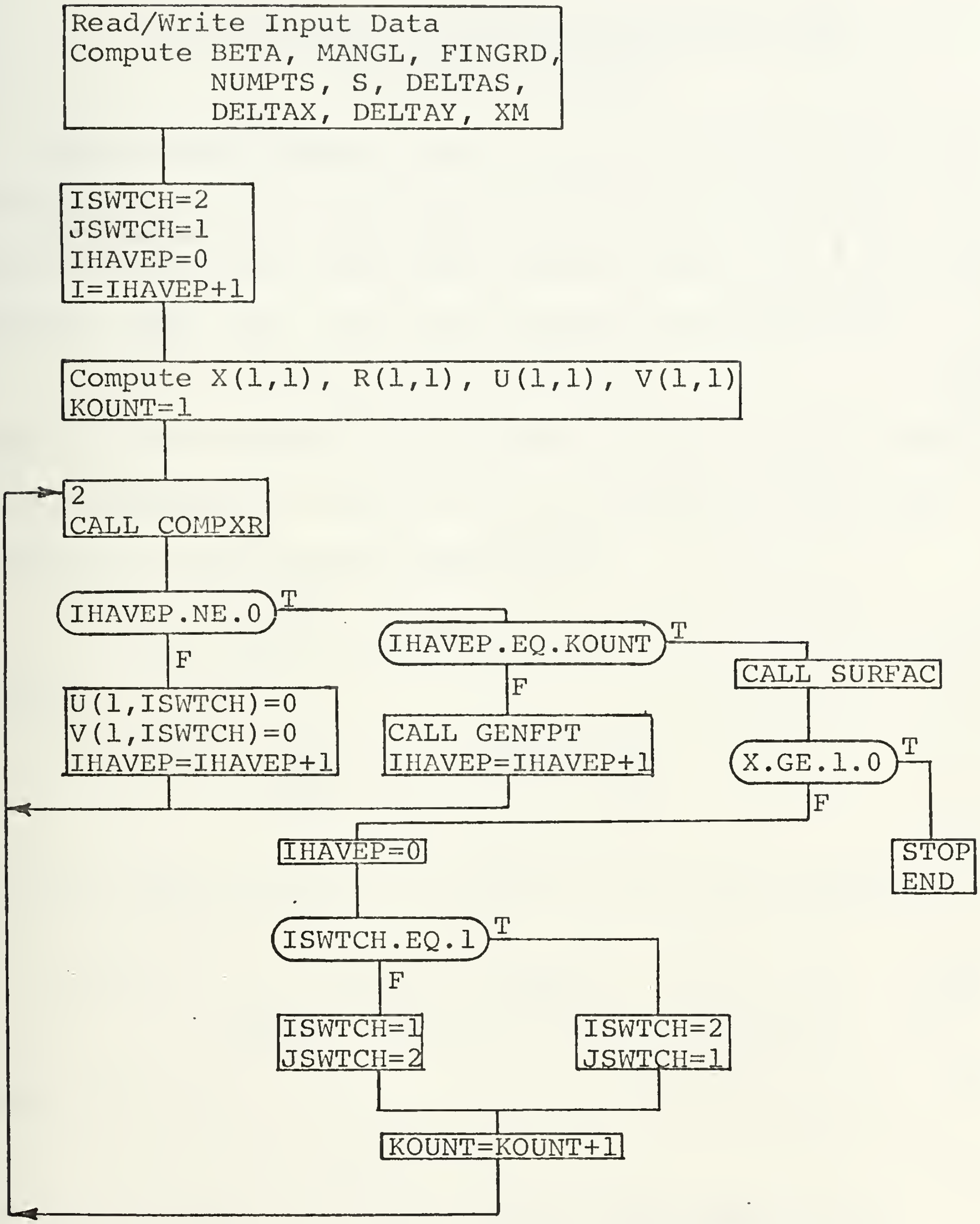
The following variables in the program take on the values defined below. All are floating point quantities unless otherwise stated. The dimensioned variables are listed first.

CP(200)	The pressure coefficient.
DELX,DELR(200)	The incremental distances between grid points on a right-running characteristic.
U(200,2)	The streamwise velocity perturbation.
V(200,2)	The product vr.
X,R(200,2)	The x and r coordinates of the grid point.
A,B	Used as factors in the equations for u and v.

BETA	The square root of the quantity M^2-1.
DELTAS	The incremental distance between adjacent grid points along the initial Mach line.
DELTAX	The change in x from one point to the next on the initial Mach line.
DELTAY	The change in r from one point to the next on the initial Mach line.
FINGRD	Floating point NGFN.
HO	The maximum amplitude of the deflection.
MACH	Free-stream Mach number.
MANGL	The Mach angle, α.
MFREQ	The axial mode number, an integer.
NGFN	The grid fineness ratio, an even integer, less than 200.
NUMPTS	The number of grid points on the initial Mach line.
PI	3.141593
RETA	r_η
RO	The radius to length ratio.
RZETA	r_ξ
S	The length of the initial Mach line.
SLP	Slope of the surface at a surface grid point.
XM	Floating point MFREQ.
XMAX	The value of x at a surface grid point.
YO	The intercept of a right-running Mach line with the r-axis.

APPENDIX F

SAUER-HEINZ FLOW DIAGRAM



APPENDIX G

SUPERSONIC FLOW PAST AN INFINITELY LONG WAVY WALL CYLINDER

Leonard and Hedgepeth (Ref. 4) gave a solution for supersonic flow past an infinitely long wavy wall cylinder. Detailed computations using this solution were carried out by Anderson (Ref. 14). For the cylinder wall deflection

$$h = h_0 \sin m\pi x \cos n\,\theta \tag{G-1}$$

where m is the number of half-waves per unit axial length and n indicates the number of circumferential waves, one obtains for the pressure distribution.

$$Cp = 2h_0 \frac{m\pi}{M} A_1 \sin(m\pi x + \Psi_1) \tag{G-2}$$

where

$$A_1 e^{i\Psi_1} = \frac{M}{\sqrt{M^2-1}} \frac{H_n^{2}\,[m\pi R(M^2-1)^{\frac{1}{2}}]}{H_n^{2\prime}[m\pi R(M^2-1)^{\frac{1}{2}}]} \tag{G-3}$$

For the actual computation equation (G-3) was rewritten as

$$A_1 e^{i\Psi_1} = \frac{J_n(\gamma) - i\,Y_n(\gamma)}{[J_{n-1}(\gamma)-J_{n+1}(\gamma)]-i[Y_{n-1}(\gamma)-Y_{n+1}(\gamma)]} \cdot \frac{2M}{\sqrt{M^2-1}} \tag{G-4}$$

where

$$\gamma = m\pi R\sqrt{M^2-1} \tag{G-5}$$

For the case n = 0 the equation was rewritten as

$$A_1 e^{i\Psi_1} = \frac{J_0(\gamma) - i\,Y_o(\gamma)}{-J_1(\gamma) + i\,Y_1(\gamma)} \cdot \frac{M}{\sqrt{M^2-1}} \tag{G-6}$$

and

$$A_1 = |A_1 e^{i\Psi_1}| \tag{G-7}$$

$$\Psi_1 = \tan^{-1}\left[\frac{\mathrm{Im}(A_1 e^{i\Psi_1})}{\mathrm{Re}(A_1 e^{i\Psi_1})}\right] \tag{G-8}$$

The program listing appears in Appendix I.

APPENDIX H

TWO BLADE SOLUTION USING THE METHOD OF SINGULARITIES

As shown by Garrick and Rubinow (Ref. 12), the velocity potential of a single blade oscillating in supersonic flow is

$$\phi(x,y) = \frac{-1}{\sqrt{M^2-1}} \int_0^{x-y\sqrt{M^2-1}} v(\xi)\, e^{-ik\frac{M^2}{M^2-1}(x-\xi)}\, J_0\left[\frac{kM}{M^2-1}\sqrt{(x-\xi)^2-(M^2-1)y^2}\right] d\xi \qquad \text{(H-1)}$$

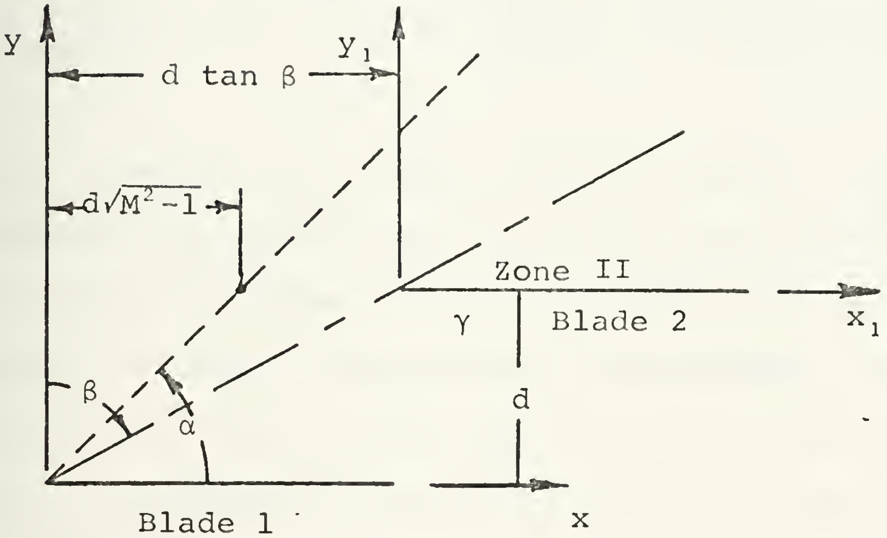


Figure H-1. Method of Singularities Configuration

The solution for supersonic flow past two oscillating blades in Zone II, (Figure H-1), can be obtained by superposition of single blade solutions. For this purpose the disturbed flow field in Zone II is written as the linear superposition of the single blade solutions for blades 1 and 2. However, in order to satisfy the flow tangency condition on the upper surface

of blade 2, a third single blade solution with unknown normal velocity must be added. Hence, one can write

$$\phi_T = \phi_1 + \phi_2 + \phi_{IP} \qquad \text{(H-2)}$$

where

$$\phi_1 = \frac{-1}{\sqrt{M^2-1}} \int_0^{x-y\sqrt{M^2-1}} v(\xi) e^{-ik\frac{M^2}{M^2-1}(x-\xi)} J_0\left[\frac{kM}{M^2-1}\sqrt{(x-\xi)^2-(M^2-1)y^2}\right] d\xi \qquad \text{(H-3)}$$

$$\phi_2 = \frac{-1}{\sqrt{M^2-1}} \int_0^{x_1-y_1\sqrt{M^2-1}} v(\xi) e^{-i\frac{kM^2}{M^2-1}(x_1-\xi)} J_0\left[\frac{kM}{M^2-1}\sqrt{(x_1-\xi)^2-(M^2-1)y_1^2}\right] d\xi \qquad \text{(H-4)}$$

$$\phi_{IP} = \frac{-1}{\sqrt{M^2-1}} \int_0^{x_1-y_1\sqrt{M^2-1}} v_\sigma(\xi) e^{-i\frac{kM^2}{M^2-1}(x_1-\xi)} J_0\left[\frac{kM}{M^2-1}\sqrt{(x_1-\xi)^2-(M^2-1)y_1^2}\right] d\xi \qquad \text{(H-5)}$$

where v_σ denotes the unknown normal velocity. Equation (H-3) represents the disturbance due to the first blade, equation (H-4) the disturbance due to the second blade alone, and equation (H-5) the interference disturbance. Note that the blade 2 and interference potentials are expressed in terms of an x_1, y_1 coordinate system with origin at the leading edge of blade 2.

Satisfying the boundary condition on the upper surface of blade 2 gives an equation for the unknown velocity, v_σ, i.e.,

$$v_\sigma(x,y) = \frac{1}{\sqrt{M^2-1}} \frac{\partial}{\partial y} \int_0^{x-y\sqrt{M^2-1}} v(\xi) e^{-i\frac{kM^2}{M^2-1}(x-\xi)} \cdot J_0\left[\frac{Mk}{M^2-1}\sqrt{(x-\xi)^2-(M^2-1)y^2}\right] d\xi \qquad \text{(H-6)}$$

at y = d.

Now setting

$$v(\xi) = ik\, Z(\xi) + Z'(\xi) \tag{H-7}$$

and assuming the blades to pitch about their leading edges, i.e.,

$$Z(\xi) = \xi \tag{H-8}$$

and then performing the differentiation, one obtains

$$\begin{aligned} v_\sigma(x,y) = \frac{1}{\sqrt{M^2-1}} \Bigg\{ & \int_0^{x-y\sqrt{M^2-1}} [ikZ(\xi)+Z'(\xi)]\, e^{-i\frac{kM^2}{M^2-1}(x-\xi)} \\ & \cdot \frac{kMy}{\sqrt{(x-\xi)^2-(M^2-1)y^2}} \cdot J_1\Big[\frac{kM}{M^2-1} \sqrt{(x-\xi)^2-(M^2-1)y^2}\Big] d\xi \\ & - [ikZ(x-y\sqrt{M^2-1})+Z'(x-y\sqrt{M^2-1})] \cdot e^{-i\frac{kM^2}{M^2-1}y} \Bigg\} \\ & \text{at } y = d \end{aligned} \tag{H-9}$$

Since, in general

$$C_P = ik\phi + \phi_x \tag{H-10}$$

the total pressure coefficient can be written as

$$C_P = C_{P1} + C_{P2} - C_{P_{IP}} \tag{H-11}$$

where C_{P1} is obtained from the single blade solution for blade 1 at $y = d$,

$$\begin{aligned} C_{P1} = \frac{k^2}{\sqrt{M^2-1}} \Bigg(& \int_0^{x-d\sqrt{M^2-1}} [-Z(\xi)+\tfrac{i}{k}Z'(\xi)]\, e^{-i\frac{kM^2}{M^2-1}(x-\xi)} \\ & \cdot J_0\Big[\frac{kM}{M^2-1} \sqrt{(x-\xi)^2-(M^2-1)d^2}\Big] d\xi - \frac{i}{k} \Bigg\{ \int_0^{x-d\sqrt{M^2-1}} [-Z'(\xi)+\tfrac{i}{k}Z''(\xi)] \\ & \cdot e^{-i\frac{kM^2}{M^2-1}(x-\xi)} \cdot J_0\Big[\frac{kM}{M^2-1}\sqrt{(x-\xi)^2-(M^2-1)d^2}\Big] d\xi \\ & + \frac{i}{k} e^{-i\frac{kM^2}{M^2-1}x} \cdot J_0\Big[\frac{kM}{M^2-1} \sqrt{x^2-(M^2-1)d^2}\Big] \Bigg\} \Bigg) \end{aligned} \tag{H-12}$$

C_{P2} from the single blade solution for blade 2 at $y_1 = 0$,

$$C_{P2} = \frac{k^2}{\sqrt{M^2-1}}\left(\int_0^{x_1} [-Z(\xi)+\frac{i}{k}Z'(\xi)]e^{-i\frac{kM^2}{M^2-1}(x_1-\xi)}\right.$$

$$\cdot\, J_0[\frac{kM}{M^2-1}(x_1-\xi)]d\xi - \frac{i}{k}\left\{\int_0^{x_1} [-Z'(\xi)+\frac{i}{k}Z''(\xi)]e^{-i\frac{kM^2}{M^2-1}(x_1-\xi)}\right.$$

$$\left.\left.\cdot\, J_0[\frac{kM}{M^2-1}(x_1-\xi)]d\xi + \frac{i}{k}\, e^{-i\frac{kM^2}{M^2-1}x_1}\; J_0[\frac{kM}{M^2-1}\, x_1]\right\}\right) \qquad \text{(H-13)}$$

and

$$C_{P_{IP}} = ik\,\phi_{IP} + \phi_{IP} \qquad \text{(H-14)}$$

where

$$\phi_{IP} = \frac{-1}{\sqrt{M^2-1}} \int_0^{x_1} v_\sigma(\gamma)e^{-i\frac{kM^2}{M^2-1}(x_1-\gamma)}\; J_0[\frac{kM}{M^2-1}(x_1-\gamma)]d\gamma \qquad \text{(H-15)}$$

and

$$\phi_{IPx} = \frac{-1}{\sqrt{M^2-1}}\left\{\int_0^{x_1} v_\sigma(\gamma)[-i\frac{kM^2}{M^2-1}\; e^{-i\frac{kM^2}{M^2-1}(x_1-\xi)}\right.$$

$$\cdot\, J_0[\frac{kM}{M^2-1}(x_1-\gamma)]d\gamma \;-\; \int_0^{x_1} v_\sigma(\gamma)\; e^{-i\frac{kM}{M^2-1}(x_1-\gamma)} \cdot \frac{kM}{M^2-1}$$

$$\left.\cdot\, J_1[\frac{kM}{M^2-1}(x_1-\gamma)]d\gamma\right\} \qquad \text{(H-16)}$$

where

$$v_\sigma(x) = \frac{1}{\sqrt{M^2-1}}\left\{\int_0^{x-d\sqrt{M^2-1}} [ikZ(\xi)+Z'(\xi)]e^{-i\frac{kM^2}{M^2-1}(x-\xi)}\right.$$

$$\cdot\, \frac{kMd}{\sqrt{(x-\xi)^2-(M^2-1)d^2}} \cdot J_1[\frac{kM}{M^2-1}\; \sqrt{(x-\xi)^2-(M^2-1)d^2}]d\xi$$

$$\left. -[ikZ(x-d\sqrt{M^2-1}) + Z'(x-d\sqrt{M^2-1})]e^{-i\frac{kM^2}{M^2-1}d}\right\} \qquad \text{(H-17)}$$

Note that the substitution of $v_\sigma(x)$ into equations (H-15) and (H-16) for ϕ_{IP} and ϕ_{IPx} requires one to relate x and γ by

$$x = \gamma + d \tan \beta \qquad \text{(H-18)}$$

(see Figure H-1)

These integrals were computed using the Trapexoidal Rule and IBM subroutine BESJ. A listing of the program follows in Appendix I.

APPENDIX I

PROGRAM LISTINGS

```
C
C           LINEARIZED METHOD OF CHARACTERISTICS
C     FOR   SUPERSONIC FLOW PAST A VIBRATING CYLINDRICAL SHELL
C
      DIMENSION DATE(3)
      COMMON DPHII,DPHIK,PHI(400,2),PHK(400,2),QPHI(400,2),IPHI1,IPHI2,
     4IKM,IKW,PARW,W,II,REALCP(400),CP(400),AMAGCP(400),
     1    MACH,K,MANGL,RADIUS,DELTAS,XLENGT,BETA,DSTSTR,HDSTRL,TRNGLH,
     2 X(400,2),R(400,2),NFREQ,FINGRD,ISWTCH,JSWTCH,ILINE,MLINE,JLINE,
     3 MFREQ,IHAVEP,KOUNT
      INTEGER FINGRD
      REAL   MACH,MANGL,K
      COMPLEX DPHII,DPHIK,PHI,IKM,CP,PX ,IPHI1,IPHI2,IKW,PHK,II,QPHI,PR
C
C     PRINT NAME OF PROGRAM AND DATE OF RUN
C
      READ (5,10)DATE
   10 FORMAT (3A4)
   11 FORMAT ('1',/////,40X,'LINEARIZED METHOD OF CHARACTERISTICS'//40X,
     1'FOR SUPERSONIC FLOW PAST A VIBRATING CYLINDRICAL SHELL'//20X
     2'DATE OF RUN-',3A4///)
      WRITE (6,11) DATE
C
C     READ INPUT
C
      READ(5,1)MACH,RADIUS,K,MFREQ,NR,NFREQ,FINGRD
    1 FORMAT(F20.8,2F10.3,4I5)
C
C     COMPUTE CONSTANTS
C
      II=(0.0,1.0)
      BETA=SQRT(MACH**2-1.0)
      MANGL=ARSIN(1.0/MACH)
      XLENGT=1./(2.*COS(MANGL))
      HEIGHT=SIN(MANGL)*XLENGT
      DELTAS=XLENGT /FINGRD
      DSTSTR=DELTAS*COS(MANGL)*2.0
      HDSTRL=.5*DSTSTR
      ML=IFIX(FINGRD+1.0)
      TRNGLH=HDSTRL*TAN(MANGL)
C
C     SET COUNTERS
C
   20 ISWTCH=2
      JSWTCH=1
      IHAVEP=1
      KCUNT=1
C
```

```
C     COMPUTE INTIAL VALUES                                             00000490
C                                                                       00000500
      I=IHAVEP                                                          00000510
      QPHI(I,JSWTCH)=(0.,0.)                                            00000520
      PHK(I,JSWTCH)=(0.,0.)                                             00000530
      X(IHAVEP,JSWTCH)=0.0                                              00000540
      R(IHAVEP,JSWTCH)=RADIUS                                           00000550
      PI=3.141593                                                       00000560
      PARW=MFREQ*PI*COS(MFREQ*PI*X(IHAVEP,JSWTCH))                      00000570
      QR=PARW                                                           00000580
      PHI(I,JSWTCH)=-2.0*QR/MACH                                        00000590
      PX=(MACH/(2.0*BETA))*(PHK(I,JSWTCH)+PHI(I,JSWTCH))                00000600
      CP(KOUNT)=-2.0*(II*K*QPHI(I,JSWTCH)+PX)                           00000610
      REALCP(KOUNT)=REAL(CP(KOUNT))                                     00000620
      AMAGCP(KOUNT)=AIMAG(CP(KOUNT))                                    00000630
C                                                                       00000640
C     WRITE OUT INPUT                                                   00000650
C                                                                       00000660
      WRITE (6,12) MACH,K,NFREQ,FINGRD ,RADIUS  ,MFREQ                  00000670
 12   FORMAT (1H0,20X,'FREESTREAM MACH NUMBER =',F9.6//20X,'REDUCED FREQ00000680
     1=',F9.5//20X,'RAD MODE NO. N    =',I2//20X,'GRID FINENESS IS = ',  00000690
     2 I3  //20X,'CYLINDER RADIUS IS =',F9.2//20X,'AXIAL MODE NO. M   ='00000700
     3,I2)                                                              00000710
      KCUNT=2                                                           00000720
C                                                                       00000730
C     COMPXR COMPUTES THE VALUE FOR X AND R GIVEN IHAVEP,ISWTCH         00000740
C                                                                       00000750
 104   CALL COMPXR                                                      00000760
      PARW=MFREQ*PI*COS(MFREQ*PI*X(IHAVEP,ISWTCH))                      00000770
      W=SIN(MFREQ*PI*X(IHAVEP,ISWTCH))                                  00000780
      XKM=(K*MACH)/BETA                                                 00000790
      IKM=II*XKM                                                        00000800
      XKW=K*W                                                           00000810
      IKW=II*XKW                                                        00000820
C                                                                       00000830
C     THE PROGRAM NOW GOES TO 30 IF POINT IS ON INITIAL MACH LINE.      00000840
C                                                                       00000850
 1001  IF (IHAVEP.EQ.1) GO TO 30                                        00000860
C                                                                       00000870
C     TEST WHETHER THIS IS A SURFACE POINT.                             00000880
C                                                                       00000890
      IF  (IHAVEP.EQ.KOUNT) GO TO 40                                    00000900
      GO TO 60                                                          00000910
C                                                                       00000920
 30    CALL MACHLN                                                      00000930
      GO TO 50                                                          00000940
C                                                                       00000950
 40   CALL PANPNT                                                       00000960
```

```
      GO TO 50
C
 60   CALL GENFPT
   50 A=IHAVEP
      B=ISWTCH
      PX=(MACH/(2.0*BETA))*(PHK(A,B)+ PHI(A,B))
      PR=(MACH/2.0)*(PHK(A,B)-PHI(A,B))
 1000   IF (IHAVEP.EQ.KOUNT) GO TO 25
      GO TO 35
C
C     INCREMENT FOR NEXT LINES
C
 25   IF (ISWTCH.EQ.1.) GO TO 103
      ISWTCH=1
      JSWTCH=2
      IHAVEP=1
      GO TO 108
 103  ISWTCH=2
      JSWTCH=1
      IHAVEP=1
      GO TO 108
C
C     AT 35 INCREMENT ALONG PRESENT LINE
C
 35   IHAVEP=IHAVEP+1
      GO TO 104
 108   KOUNT=KOUNT+1
      IF ((KOUNT-1).EQ.ML)GO TO 101
      GO TO 104
  101 CONTINUE
      CALL  INTEG(REALCP,MFREQ,NR,DSTSTR,ML)
      STOP
      END
C


      SUBROUTINE COMPXR
C
      REAL   MACH,MANGL,K,KW
C
C     COMPUTES THE X AND R COORDINATES
C
      COMMON DPHII,DPHIK,PHI(400,2),PHK(400,2),QPHI(400,2),IPHI1,IPHI2,
     4IKM,IKW,PARW,W,II,REALCP(400),CP(400),AMAGCP(400),
     1    MACH,K,MANGL,RADIUS,DELTAS,XLENGT,BETA,DSTSTR,HDSTRL,TRNGLH,
     2  X(400,2),R(400,2),NFREQ,FINGRD,ISWTCH,JSWTCH,ILINE,MLINE,JLINE,
     3  MFREQ,IHAVEP,KOUNT
      COMPLEX DPHII,DPHIK,PHI,IKM,CP,PX ,IPHI1,IPHI2,IKW,PHK,II,QPHI,PR
```

```
      INTEGER FINGRD
C
C     TEST FOR POINT ON MACH LINE
C
      IF (IHAVEP.EQ.1) GO TO 10
      I=IHAVEP
      J=IHAVEP-1
      X(I,ISWTCH)=X(J     ,ISWTCH)+HDSTRL
      R(I,ISWTCH)=R(J     ,ISWTCH)        -TRNGLH
      RETURN
 10   I=IHAVEP
      X(I,ISWTCH)=X(I     ,JSWTCH)+HDSTRL
      R(I,ISWTCH)=R(I     ,JSWTCH)+TRNGLH
      RETURN
      END


      SUBROUTINE MACHLN
      CCMMON DPHII,DPHIK,PHI(400,2),PHK(400,2),QPHI(400,2),IPHI1,IPHI2,
     4IKM,IKW,PARW,W,II,REALCP(400),CP(400),AMAGCP(400),
     1    MACH,K,MANGL,RADIUS,DELTAS,XLENGT,BETA,DSTSTR,HDSTRL,TRNGLH,
     2  X(400,2),R(400,2),NFREQ,FINGRD,ISWTCH,JSWTCH,ILINE,MLINE,JLINE,
     3  MFREQ,IHAVEP,KOUNT
      REAL   MACH,MANGL,K,KW
      COMPLEX                 AA2,
     1A,B,C,D,DPHII,DPHIK,PHI,IKM,CP,PX ,IPHI1,IPHI2,IKW,PHK,II,QPHI
     2 ,PR
      INTEGER FINGRD
      I=IHAVEP
      PHK(I,ISWTCH)=(0.,0.)
      A=1.+DELTAS/(4.*R(I,ISWTCH)*MACH)+0.5*IKM*DELTAS
      AA2=1.-DELTAS/ (4.*R(I,JSWTCH)*MACH)-0.5*IKM*DELTAS
      PHI(I,ISWTCH)=PHI(I,JSWTCH)*(AA2/A)
      QPHI(I,ISWTCH)=(0.,0.)
 10   RETURN
      END


      SUBROUTINE GENFPT
C
C     GENFPT COMPUTES THE VELOCITY POTENTIAL AND ITS DERIVATIVES AT A
C     GENERAL FLOW FIELD POINT.
C
      CCMMON DPHII,DPHIK,PHI(400,2),PHK(400,2),QPHI(400,2),IPHI1,IPHI2,
     4IKM,IKW,PARW,W,II,REALCP(400),CP(400),AMAGCP(400),
     1    MACH,K,MANGL,RADIUS,DELTAS,XLENGT,BETA,DSTSTR,HDSTRL,TRNGLH,
     2  X(400,2),R(400,2),NFREQ,FINGRD,ISWTCH,JSWTCH,ILINE,MLINE,JLINE,
     3  MFREQ,IHAVEP,KOUNT
```

```
      REAL    MACH,MANGL,K,KW,N
      CCMPLEX                   AA1,AA2,AA3,
     1A,B,C,D,DPHII,DPHIK,PHI,IKM,CP,PX ,IPHI1,IPHI2,IKW,PHK,II,QPHI
     2 ,PR,E,F
      INTEGER FINGRD
C
      I=IHAVEP
      J=IHAVEP -1
      N=FLOAT(NFREQ)
C
      A=1.-DELTAS/(4.*R(I,ISWTCH)*MACH)+0.5*IKM*DELTAS
C
      AA1=1.+DELTAS/(4.*R(J,ISWTCH)*MACH)-0.5*IKM*DELTAS
C
      AA3=-1.+DELTAS/ (4.*R(I,JSWTCH)*MACH)+0.5*IKM*DELTAS
C
      B=DELTAS/(4.*R(I,ISWTCH)*MACH)+0.5*IKM*DELTAS-0.25*DELTAS**2*
     1  (K**2-(N**2/(R(I,ISWTCH)**2*MACH**2)))
C
C
      C=PHK(J,ISWTCH)*AA1+PHI(J,ISWTCH)*(-DELTAS/(4.*R(J,ISWTCH)*MACH)
     1 -0.5*IKM*DELTAS+0.25*DELTAS**2*(K**2-(N**2/(R(I,ISWTCH)**2*MACH**
     22))))+ QPHI(J,ISWTCH)*DELTAS*0.5*(K**2- N**2/(R(J,ISWTCH)**2*MACH
     3**2))+   QPHI(J,ISWTCH)*DELTAS*0.5*(K**2- N**2/(R(I,ISWTCH)**2*
     4 MACH**2))
C
C
      D=DELTAS/(4.0  *R(I,ISWTCH)*MACH)-0.5  *IKM*DELTAS+0.25  *DELTAS**
     12*(K**2-(N**2/(R(I,ISWTCH)**2*MACH**2)))
C
C
      F=PHI(I,JSWTCH)*AA3+PHK(I,JSWTCH)*(-DELTAS/(4.*R(I,JSWTCH)*MACH)
     1 +0.5*IKM*DELTAS-0.25*DELTAS**2*(K**2-(N**2/(R(I,ISWTCH)**2*MACH**
     22))))- QPHI(I,JSWTCH)*DELTAS*0.5*(K**2- N**2/(R(I,JSWTCH)**2*MACH
     3**2))-   QPHI(I,JSWTCH)*DELTAS*0.5*(K**2- N**2/(R(I,ISWTCH)**2*
     4 MACH**2))
C
      E=-1.-DELTAS/(4.*R(I,ISWTCH)*MACH)-0.5*IKM*DELTAS
C
C     CRAMER'S RULE
C
      PHK(I,ISWTCH)=(C*E-B*F)/(A*E -B*D)
      PHI(I,ISWTCH)=(F*A-C*D)/(A*E -B*D)
C
C     TRAPAZOIDAL RULE TO INTEGRATE.
C
      IPHI1=QPHI(I,JSWTCH)+0.5*(PHK(I,JSWTCH)+PHK(I,ISWTCH))*DELTAS
      IPHI2=QPHI(J,ISWTCH)+0.5*(PHI(J,ISWTCH)+PHI(I,ISWTCH))*DELTAS
```

```
C
C     AVERAGE THE TWO
C
      QPHI(I,ISWTCH)=0.5*(IPHI1+IPHI2)
 15   RETURN
      END


      SUBROUTINE PANPNT
C
C     PANPNT COMPUTES THE VALUES OF PHI'S AND QHPI AT A SURFACE POINT
C
      CCMMON DPHII,DPHIK,PHI(400,2),PHK(400,2),QPHI(400,2),IPHI1,IPHI2,
     4IKM,IKW,PARW,W,II,REALCP(400),CP(400),AMAGCP(400),
     1    MACH,K,MANGL,RADIUS,DELTAS,XLENGT,BETA,DSTSTR,HDSTRL,TRNGLH,
     2  X(400,2),R(400,2),NFREQ,FINGRD,ISWTCH,JSWTCH,ILINE,MLINE,JLINE,
     3  MFREQ,IHAVEP,KOUNT
      REAL    MACH,MANGL,K,KW,N
      COMPLEX                     AA1,AA2
     1A,B,C,D,DPHII,DPHIK,PHI,IKM,CP,PX ,IPHI1,IPHI2,IKW,PHK,II,QPHI
     2 ,PR
      INTEGER FINGRD
      N=FLOAT(NFREQ)
      I=IHAVEP
      J=IHAVEP-1
      A=1.-DELTAS/(4.*R(I,ISWTCH)*MACH)+0.5*IKM*DELTAS
      AA1=1.+DELTAS/(4.*R(J,ISWTCH)*MACH)-0.5*IKM*DELTAS
      B=DELTAS/(4.*R(I,ISWTCH)*MACH)+0.5*IKM*DELTAS-0.25*DELTAS**2*
     1  (K**2-(N**2/(R(I,ISWTCH)**2*MACH**2)))
      C=PHK(J,ISWTCH)*AA1+PHI(J,ISWTCH)*(-DELTAS/(4.*R(J,ISWTCH)*MACH)
     1 -0.5*IKM*DELTAS+0.25*DELTAS**2*(K**2-(N**2/(R(I,ISWTCH)**2*MACH**
     22))))+ QPHI(J,ISWTCH)*DELTAS*0.5*(K**2- N**2/(R(J,ISWTCH)**2*MACH
     3**2))+   QPHI(J,ISWTCH)*DELTAS*0.5*(K**2- N**2/(R(I,ISWTCH)**2*
     4 MACH**2))
C
C
      PHK(I,ISWTCH)=       (C+B*(2.0/MACH)*(PARW+IKW))/(A+B)
      PHI(I,ISWTCH)=       (-A*(2.0/MACH)*(PARW+IKW)+C)/(A+B)
C
      QPHI(I     ,ISWTCH)=QPHI(J,ISWTCH)+0.5*(PHI(J,ISWTCH)+PHI(I     ,
     1  ISWTCH))*DELTAS
      PX=(MACH/(2.0*BETA))*(PHK(I,ISWTCH)+PHI(I,ISWTCH))
      CP(KOUNT)=-2.0*(II*K*QPHI(I,ISWTCH)+PX)
      REALCP(KOUNT)=REAL(CP(KOUNT))
      AMAGCP(KOUNT)=AIMAG(CP(KOUNT))
 10   RETURN
      END
```

```
      SUBROUTINE INTEG(Y,M,NR,DX,ML)
      DIMENSION Y(ML)
      Q=0.0
      X1=0.0
      RN=FLOAT(NR)
      PSI1=0.0
      PROD1=0.0
      DO 1 I=2,ML
      X2=X1+DX
      PSI2=SIN(RN*3.141593*X2)
      PROD2=PSI2*Y(I)
      Q=Q+(PROD2+PROD1)*DX/2.0
      PROD1=PROD2
      X1=X2
    1 CONTINUE
      Q=Q/2.0
      WRITE(6,100)M,NR,Q
  100 FORMAT(' ',10X,'Q(',I2,',',I2,')R=',E13.7,///)
  101 FORMAT('0','GENERALIZED AERODYNAMIC FORCE:',/)
      RETURN
      END
```

```
C
C                     ONE DEGREE OF FREEDOM CASCADE
C
      CIMENSION DATE(3)
      DIMENSION U22R(400,2),U22I(400,2),V22R(400,2),V22I(400,2),C22R(400
     1,2),C22I(400,2),X(400,2),Y(400,2)
      DIMENSION U33R(400,2),U33I(400,2),V33R(40C,2),V33I(400,2),C33R(400
     1,2),C33I(400,2)
      COMMON/BLK1/NGRDFN,FSTRMN,REDFRQ,XSUBO,TNWDST,FMANGL,XLNGTH,DELTAS
     1,DSTSTR,HDSTRL,TRNGLH,U22R,U22I,V22R,V22I,C22R,C22I,X,Y,S,DELTA,
     2ISWTCH,JSWTCH,JLINE,IHAVEP,AI,BI,K12R,K12I,K34R,K34I,K56R,K56I,
     3VRPANL,VIPANL,STGANG,NSTPTS,LCOUNT,STAG
      CCMMON/BLK5/U33R,U33I,V33R,V33I,C33R,C33I
      CCMMON/PCOR/ICO
      COMMON/STG/STGR
      CCMMON/BLADE/NUM,NUMBLD
      COMMON/BLK6/W
      REAL K12R,K12I,K34R,K34I,K56R,K56I
      INTEGER OLDL
C
C     PRINT NAME OF PROGRAM/DATE OF RUN.
C
      READ(5,100)DATE
  100 FCRMAT(3A4)
      WRITE(6,101)DATE
  101 FORMAT(1H1,14(/),14(/),37X,51H OSCILLATING CASCADE PROGRAM
     1    ---RUN OF---,3A4,/,1H1)
C
C     PRINT OUT ALL INPUT INFORMATION VIA INPUT.
C
      CALL INPUT
      OLDL=0
      ICC=0
C
C     INITIALIZE FLOW QUANTITIES AND LOGIC VARIABLES.
C
      CALL INTIAL
      FAC=1.0/DSTSTR
      NEWDST=FAC
      XX=FAC-NEWDST
      IF(XX.GE.0.5)NEWDST=NEWDST+1
      LIMIT=(NUMBLD-1)*NSTPTS+NEWDST
      LIMW=NEWDST
      MAXI=NGRDFN+NSTPTS
      NUM=0
      NUMOLD=0
      NUMPI=1
 1000 CCNTINUE
```

```
      CALL COMPXY
C
C     IF POINT IS ON THE INITIAL RIGHT-RUNNING MACH LINE, CALL MACHLN.
C
      IF(LCOUNT.EQ.0)GO TO 1
C
C     IF POINT IS ON A BLADE OR IN A WAKE, GO TC 2.
C
      IF(IHAVEP.EQ.LCOUNT)GO TO 2
C
C     IF POINT IS ON INITIAL LEFT-RUNNING MACH LINE, CALL MACHLN.
C
      IF(IHAVEP.EQ.0)GO TO 1
C
C     CONDITICN FOR FIRST ENCOUNTER WITH A BLADE.
C
      IF((IHAVEP.EQ.NUMPI*NSTPTS).AND.(JLINE.EQ.MAXI))GO TO 7
      CALL GENFPT
      IF(IHAVEP.EQ.LIMIT)GO TO 5
      IHAVEP=IHAVEP+1
      GO TO 1000
    1 CALL MACHLN
      IF(IHAVEP.EQ.LIMIT)GO TO 5
      IHAVEP =IHAVEP+1
      GO TO 1000
    2 ICC=1
C
C     IF POINT IS IN A WAKE, CALL WAKE.
C
      IF(IHAVEP.GE.LIMW)GC TO 4
      CALL TOP
      CALL BOTTCM
      IF(IHAVEP.EQ.LIMIT)GO TO 8
      IHAVEP=IHAVEP+1
      IF(NUM.EQ.0)GO TO 1000
      NUM=NUM-1
      LCOUNT=LCOUNT+NGRDFN
      LIMW=LIMW-NSTPTS
      GO TO 1000
    4 CALL WAKE
      IF(IHAVEP.EQ.LIMIT)ICO=0
C
C     IF THIS IS THE LAST POINT TO BE COMPUTED IN THE ENTIRE FLOW
C     FIELD AREA, TERMINATE PROGRAM.
C
      IF((NUM.GE.NUMBLD-1).AND.(IHAVEP.EQ.LIMIT))GO TO 8
      IF(IHAVEP.EQ.LIMIT)GO TO 5
      IHAVEP=IHAVEP+1
```

```
      IF(NUM.EQ.0)GO TO 1000                                            00000970
      LCOUNT=LCOUNT+NGRDFN                                              00000980
      NUM=NUM-1                                                         00000990
      GO TO 1000                                                        00001000
    5 CONTINUE                                                          00001010
C                                                                       00001020
C     SWITCH LOGIC VARIABLES TO START THE COMPUTATION BACK AT THE       00001030
C     INITIAL LEFT-RUNNING MACH LINE.                                   00001040
C                                                                       00001050
      IHAVEP=0                                                          00001060
      NUM=NUMOLD                                                        00001070
      JLINE=JLINE+1                                                     00001080
      IF(ISWTCH.EQ.1)GO TO 9                                            00001090
      ISWTCH=1                                                          00001100
      JSWTCH=2                                                          00001110
      GO TO 10                                                          00001120
    9 ISWTCH=2                                                          00001130
      JSWTCH=1                                                          00001140
   10 CONTINUE                                                          00001150
      IF(NUMOLD.GT.0)GO TO 6                                            00001160
      LCOUNT=LCOUNT+1                                                   00001170
      GO TO 1000                                                        00001180
    6 LCOUNT=OLDL                                                       00001190
      LIMW=NUMOLD*NSTPTS+NEWDST                                         00001200
      OLDL=OLDL+1                                                       00001210
      GO TO 1000                                                        00001220
    7 CALL NEWBLD                                                       00001230
      ICO=1                                                             00001240
      NUMOLD=NUM                                                        00001250
      LCOUNT= IHAVEP+NGRDFN                                             00001260
      JLINE=IHAVEP                                                      00001270
      IHAVEP=IHAVEP+1                                                   00001280
      OLDL=IHAVEP                                                       00001290
      NUMPI=NUM+1                                                       00001300
      MAXI=NGRDFN+NUMPI*NSTPTS                                          00001310
      NUM=NUM-1                                                         00001320
      GO TO 1000                                                        00001330
    8 CONTINUE                                                          00001340
      STOP                                                              00001350
      END                                                               00001360


      SUBROUTINE INPUT                                                  00001370
C                                                                       00001380
C     SUBROUTINE INPUT READS ALL INPUT.                                 00001390
C                                                                       00001400
      DIMENSION U22R(400,2),U22I(400,2),V22R(400,2),V22I(400,2),C22R(40000001410
     1,2),C22I(400,2),X(400,2),Y(400,2)                                 00001420
```

```
      CCMMON/BLK1/NGRDFN,FSTRMN,REDFRQ,XSUBO,TNWCST,FMANGL,XLNGTH,DELTAS
     1,DSTSTR,HDSTRL,TRNGLH,U22R,U22I,V22R,V22I,C22R,C22I,X,Y,S,DELTA,
     2ISWTCH,JSWTCH,JLINE,IHAVEP,AI,BI,K12R,K12I,K34R,K34I,K56R,K56I,
     3VRPANL,VIPANL,STGANG,NSTPTS,LCOUNT,STAG
      CCMMON/STG/STGR
      COMMON/BLADE/NUM,NUMBLD
      CCMMON/BLK6/W
      REAL K12R,K12I,K34R,K34I,K56R,K56I
      NAMELIST/NAM1/NGRDFN,FSTRMN,REDFRQ,XSUBO,TNWDST,STGANG,FAZE,NUMBLD
      READ(5,NAM1)
      WRITE(6,NAM1)
      FMANGL=ARSIN(1.0/FSTRMN)
      XLNGTH=TNWDST/SIN(FMANGL)
      FNGDPT=FLOAT(NGRDFN)
      DELTAS=XLNGTH/FNGDPT
      TRNGLH=TNWDST/FNGDPT
      HDSTRL=DELTAS*COS(FMANGL)
      DSTSTR=HDSTRL*2.0
      STAG=TNWDST*TAN(STGANG*0.1745329E-01)
      STGR=STAG-(TNWDST/TAN(FMANGL))
      NSTPTS=STGR/DSTSTR
      IF(NSTPTS.LT.1)NSTPTS=1
      STGR=NSTPTS*DSTSTR
      STGANG=ATAN((STGR+(TNWDST/TAN(FMANGL)))/TNWCST)*57.29578
      STAG=STGR+(TNWDST/TAN(FMANGL))
      S=SQRT(FSTRMN*FSTRMN-1.0)
      W=FSTRMN*FSTRMN/(FSTRMN*FSTRMN-1.0)
      DELTA=FAZE*0.1745329E-01
      WRITE(6,5)NGRDFN,FSTRMN,REDFRQ,TNWDST,XSUBO,STGANG,DSTSTR,FAZE,NUM
     1BLD
    5 FORMAT(////,28H GRID FINENESS INPUT NUMBER=,I10,///,24H FREESTREAM
     1 MACH NUMBER=,F20.7,///,34H REDUCED(DIMENSICNLESS) FREQUENCY=,F20.
     27,///,27H DISTANCE BETWEEN AIRFCILS=,F20.7,///,52H HCRIZONTAL POSI
     3TION(DIMENSIONLESS) OF ELASTIC AXIS=,F20.7,///,26H COMPATABLE STAG
     4GER ANGLE=,F20.7,///,9H DELTA X=,F20.7,///25H UPPER AIRFOIL PHASE
     5LAG=,F20.7,///,1X,I2,' BLADES',/)
      WRITE(6,6)
    6 FORMAT('1',10X,'VALUES OF TEIPEL COMPLEX PERTURBATICN AMPLITUCES',
     1///,1X,'BLD',T10,'X',T21,'UR',T42,'UI',T58,'VR',T73,'VI',T90,'CR',
     2T107,'CI',/)
      RETURN
      END
```

```
      SUBROUTINE INTIAL
C
C     INTIAL INITIALIZES ALL FLOW FIELD QUANTITIES.
C
      DIMENSICN U22R(400,2),U22I(400,2),V22R(400,2),V22I(400,2),C22R(400
     1,2),C22I(400,2),X(400,2),Y(400,2)
      DIMENSICN U33R(400,2),U33I(400,2),V33R(400,2),V33I(400,2),C33R(400
     1,2),C33I(400,2)
      CCMMON/BLK1/NGRDFN,FSTRMN,REDFRQ,XSUBO,TNWCST,FMANGL,XLNGTH,DELTAS
     1,DSTSTR,HDSTRL,TRNGLH,U22R,U22I,V22R,V22I,C22R,C22I,X,Y,S,DELTA,
     2ISWTCH,JSWTCH,JLINE,IHAVEP,AI,BI,K12R,K12I,K34R,K34I,K56R,K56I,
     3VRPANL,VIPANL,STGANG,NSTPTS,LCOUNT,STAG
      CCMMON/BLK5/U33R,U33I,V33R,V33I,C33R,C33I
      CCMMON/BLK6/W
      CCMMON/BLADE/NUM,NUMBLD
      REAL K12R,K12I,K34R,K34I,K56R,K56I
      AI=.5*REDFRQ*DSTSTR
      BI=AI*W/2.0
C
C     SET UP INITIAL VALUES FOR U,V,AND C AT (0,C).
C
   50 VRPANL=-1.0/S
      VIPANL=REDFRQ*XSUBO/S
 60   U22R(1,1)=-VRPANL
      U22I(1,1)=-VIPANL
      V22R(1,1)=-U22R(1,1)
      V22I(1,1)=-U22I(1,1)
      C22R(1,1)=-U22R(1,1)
      C22I(1,1)=-U22I(1,1)
      X(1,1)=0.0
      Y(1,1)=0.0
      U33R(1,1)=VRPANL
      U33I(1,1)=VIPANL
      V33R(1,1)=VRPANL
      V33I(1,1)=VIPANL
      C33R(1,1)=-VRPANL
      C33I(1,1)=-VIPANL
      ISWTCH=1
      JSWTCH=2
      JLINE=0
      IHAVEP=1
      LCOUNT=0
      WRITE(6,100)X(1,1),U22R(1,1),U22I(1,1),V22R(1,1),V22I(1,1),C22R(1,
     11),C22I(1,1)
      WRITE(6,100)X(1,1),U33R(1,1),U33I(1,1),V33R(1,1),V33I(1,1),C33R(1,
     11),C33I(1,1)
  100 FORMAT(2H I,7E16.7,/)
      RETURN
```

```
      END
      SLBROUTINE MACHLN
C
C     MACHLN COMPUTES THE VALUES OF U,V,AND C ALONG THE INITIAL MACH
C     LINE AT THE GIVEN X.
C
      DIMENSICN U22R(400,2),U22I(400,2),V22R(40C,2),V22I(400,2),C22R(400
     1,2),C22I(400,2),X(400,2),Y(400,2)
      CCMMON/BLK1/NGRDFN,FSTRMN,REDFRQ,XSUBO,TNWDST,FMANGL,XLNGTH,DELTAS
     1,DSTSTR,HDSTRL,TRNGLH,U22R,U22I,V22R,V22I,C22R,C22I,X,Y,S,DELTA,
     2ISWTCH,JSWTCH,JLINE,IHAVEP,AI,BI,K12R,K12I,K34R,K34I,K56R,K56I,
     3VRPANL,VIPANL,STGANG,NSTPTS,LCOUNT,STAG
      CCMMON/BLK6/W
      REAL K12R,K12I,K34R,K34I,K56R,K56I
      I=IHAVEP+1
      ARG=REDFRQ*W*X(I,ISWTCH)
      U=COS(ARG)
      V=SIN(ARG)
C
C     IF POINT IS ON INITIAL RIGHT-RUNNING MACH LINE, GO TO 2.
C
      IF(IHAVEP.NE.0)GO TO 2
      U22R(1,ISWTCH)=-VRPANL*U-VIPANL*V
      U22I(1,ISWTCH)=-VIPANL*U+VRPANL*V
      V22R(1,ISWTCH)=-U22R(1,ISWTCH)
      V22I(1,ISWTCH)=-U22I(1,ISWTCH)
      C22R(1,ISWTCH)=-U22R(1,ISWTCH)
      C22I(1,ISWTCH)=-U22I(1,ISWTCH)
      RETURN
    2 U22R(I,ISWTCH)=VRPANL*U+VIPANL*V
      U22I(I,ISWTCH)=VIPANL*U-VRPANL*V
      V22R(I,ISWTCH)=U22R(I,ISWTCH)
      V22I(I,ISWTCH)=U22I(I,ISWTCH)
      C22R(I,ISWTCH)=-U22R(I,ISWTCH)
      C22I(I,ISWTCH)=-U22I(I,ISWTCH)
      RETURN
      END
      SLBRCUTINE GENFPT
C
C     GENFPT COMPUTES U,V,AND C AT A GENERAL FIELD POINT.
C
      DIMENSION U22R(400,2),U22I(400,2),V22R(400,2),V22I(400,2),C22R(400
     1,2),C22I(400,2),X(400,2),Y(400,2)
      DIMENSION U33R(400,2),U33I(400,2),V33R(40C,2),V33I(400,2),C33R(400
```

```
     1,2),C33I(400,2)
      CCMMON/BLK1/NGRDFN,FSTRMN,REDFRQ,XSUBO,TNWCST,FMANGL,XLNGTH,DELTAS
     1,DSTSTR,HDSTRL,TRNGLH,U22R,U22I,V22R,V22I,C22R,C22I,X,Y,S,DELTA,
     2ISWTCH,JSWTCH,JLINE,IHAVEP,AI,BI,K12R,K12I,K34R,K34I,K56R,K56I,
     3VRPANL,VIPANL,STGANG,NSTPTS,LCOUNT,STAG
      CCMMON/PCOR/ICO
      COMMON/BLK5/U33R,U33I,V33R,V33I,C33R,C33I
      REAL K12R,K12I,K34R,K34I,K56R,K56I
      I=IHAVEP+1
C
C     'ICO' INSURES PROPER SELECTION OF FLOW QUANTITIES AT P21 IF GRID
C     PCINT IS THE FIRST GRID POINT AFTER A BLADE GRID POINT.
C
      IF(ICO.EQ.1)GO TO 1
      GC TO 2
    1 A=U33R(IHAVEP,ISWTCH)
      B=U33I(IHAVEP,ISWTCH)
      C=C33R(IHAVEP,ISWTCH)
      D=C33I(IHAVEP,ISWTCH)
      ICO=0
      GO TO 10
    2 A=U22R(IHAVEP,ISWTCH)
      B=U22I(IHAVEP,ISWTCH)
      C=C22R(IHAVEP,ISWTCH)
      D=C22I(IHAVEP,ISWTCH)
   10 K12R=U22R(IHAVEP,JSWTCH)+C22R(IHAVEP,JSWTCH)+AI*U22I(IHAVEP,JSWTCH
     1)
      K12I=-AI*U22R(IHAVEP,JSWTCH)+U22I(IHAVEP,JSWTCH)+C22I(IHAVEP,JSWTC
     1H)
      K34R=U22R(IHAVEP+1,JSWTCH)-V22R(IHAVEP+1,JSWTCH)+BI*(U22I(IHAVEP+1
     1,JSWTCH)-C22I(IHAVEP+1,JSWTCH))
      K34I=U22I(IHAVEP+1,JSWTCH)-V22I(IHAVEP+1,JSWTCH)-BI*(U22R(IHAVEP+1
     1,JSWTCH)-C22R(IHAVEP+1,JSWTCH))
      K56R=A+V22R(IHAVEP,ISWTCH)+BI*(B-D)
      K56I=B+V22I(IHAVEP,ISWTCH)-BI*(A-C)
      G1=.5*(K34R+K56R)
      G2=BI*K12I
      G3=1.0-AI*BI
      G4=.5*(K34I+K56I)
      G5=BI*K12R
      G6=2.0*BI
      G7=G3*G3+G6*G6
      G8=G1-G2
      G9=G4+G5
      U22R(I,ISWTCH)=(G8*G3+G9*G6)/G7
      U22I(I,ISWTCH)=(-G8*G6+G9*G3)/G7
      V22R(I,ISWTCH)=.5*(K56R-K34R)
      V22I(I,ISWTCH)=.5*(K56I-K34I)
```

```
      C22R(I,ISWTCH)=K12R-U22R(I,ISWTCH)+U22I(I,ISWTCH)*AI
      C22I(I,ISWTCH)=K12I-U22I(I,ISWTCH)-U22R(I,ISWTCH)*AI
      RETURN
      END

      SLBROUTINE TOP
C
C     TCP COMPUTES U,V, AND C AT AN UPPER SURFACE POINT.
C
      DIMENSION U22R(400,2),U22I(400,2),V22R(400,2),V22I(400,2),C22R(400
     1,2),C22I(400,2),X(400,2),Y(400,2)
      CCMMON/BLK1/NGRDFN,FSTRMN,REDFRQ,XSUBO,TNWDST,FMANGL,XLNGTH,DELTAS
     1,DSTSTR,HDSTRL,TRNGLH,U22R,U22I,V22R,V22I,C22R,C22I,X,Y,S,DELTA,
     2ISWTCH,JSWTCH,JLINE,IHAVEP,AI,BI,K12R,K12I,K34R,K34I,K56R,K56I,
     3VRPANL,VIPANL,STGANG,NSTPTS,LCOUNT,STAG
      CCMMON/BLADE/NUM,NUMBLD
      REAL K12R,K12I,K34R,K34I,K56R,K56I
      XN=FLOAT(NUM)
      J=IHAVEP
      I=IHAVEP+1
      XNEW=X(I,ISWTCH)-STAG*XN
C
C     IF THIS IS THE FIRST BLADE OR IF THE INTERBLADE PHASE ANGLE IS
C     ZERO, SIMPLIFY THE NORMAL VELOCITY COMPUTATION.
C
      IF((DELTA.LE.1.0E-05).OR.(NUM .EQ.0))GO TO 999
      ARG=DELTA*XN
      V22R(I,ISWTCH)=-(COS(ARG)-(XNEW-XSUBO)*REDFRQ*SIN(ARG))/S
      V22I(I,ISWTCH)=-(SIN(ARG)+(XNEW-XSUBO)*RECFRQ*COS(ARG))/S
      GO TO 90
  999 V22R(I,ISWTCH)=-1.0/S
      V22I(I,ISWTCH)=(-REDFRQ*(XNEW-XSUBO))/S
 90   K34R=V22R(I     ,ISWTCH)
      K34I=V22I(I     ,ISWTCH)
    1 K12R=U22R(J     ,JSWTCH)+C22R(J     ,JSWTCH)+AI*U22I(J     ,JSWTCH)
      K12I=U22I(J,JSWTCH)+C22I(J,JSWTCH)-AI*U22R(J,JSWTCH)
      K56R=U22R(J     ,ISWTCH)+V22R(J     ,ISWTCH)+BI*(U22I(J     ,ISWTCH)-
     1C22I(J     ,ISWTCH))
      K56I=U22I(J     ,ISWTCH)+V22I(J     ,ISWTCH)-BI*(U22R(J     ,ISWTCH)-
     1C22R(J     ,ISWTCH))
      G1=1.0-AI*BI
      G2=2.0*BI
      G3=G1*G1+G2*G2
      G4=K56R-K34R-BI*K12I
      G5=K56I-K34I+BI*K12R
      U22R(I     ,ISWTCH)=(G4*G1+G5*G2)/G3
      U22I(I     ,ISWTCH)=(-G4*G2+G5*G1)/G3
```

```
      C22R(I     ,ISWTCH)=K12R-U22R(I     ,ISWTCH)+AI*U22I(I,ISWTCH)    00003710
      C22I(I     ,ISWTCH)=K12I-U22I(I     ,ISWTCH)-AI*U22R(I,ISWTCH)    00003720
  147 WRITE(6,145)NUM,XNEW,U22R(I,ISWTCH),U22I(I,ISWTCH),V22R(I,ISWTCH),00003730
     1V22I(I,ISWTCH),C22R(I,ISWTCH),C22I(I,ISWTCH)                      00003740
  145 FORMAT(2H T,I2,7E16.7,/)                                          00003750
      RETURN                                                            00003760
      END                                                               00003770

      SUBROUTINE COMPXY                                                 00003780
C                                                                       00003790
C     COMPXY COMPUTES THE GRID POINT'S X,Y POSITION.                    00003800
C                                                                       00003810
      DIMENSION U22R(400,2),U22I(400,2),V22R(400,2),V22I(400,2),C22R(40000003820
     1,2),C22I(400,2),X(400,2),Y(400,2)                                 00003830
      COMMON/BLK1/NGRDFN,FSTRMN,REDFRQ,XSUBO,TNWDST,FMANGL,XLNGTH,DELTAS00003840
     1,DSTSTR,HDSTRL,TRNGLH,U22R,U22I,V22R,V22I,C22R,C22I,X,Y,S,DELTA,  00003850
     2ISWTCH,JSWTCH,JLINE,IHAVEP,AI,BI,K12R,K12I,K34R,K34I,K56R,K56I,   00003860
     3VRPANL,VIPANL,STGANG,NSTPTS,LCOUNT,STAG                           00003870
      REAL K12R,K12I,K34R,K34I,K56R,K56I                                00003880
      I=IHAVEP+1                                                        00003890
C                                                                       00003900
C     CHECK WHETHER GRID POINT IS ON THE INITIAL LEFT-RUNNING MACH LINE.00003910
C                                                                       00003920
      IF((IHAVEP.EQ.0).AND.(LCOUNT.NE.0))GO TO 1                        00003930
      X(I,ISWTCH)=X(IHAVEP,ISWTCH)+HDSTRL                               00003940
      Y(I,ISWTCH)=Y(IHAVEP,ISWTCH)-TRNGLH                               00003950
      RETURN                                                            00003960
C                                                                       00003970
C                                                                       00003980
    1 X(I,ISWTCH)=X(1,JSWTCH)+HDSTRL                                    00003990
      Y(I,ISWTCH)=Y(1,JSWTCH)+TRNGLH                                    00004000
      RETURN                                                            00004010
      END                                                               00004020

      SUBROUTINE NEWBLD                                                 00004030
C                                                                       00004040
C     NEWBLD COMPUTES U,V, AND C AT THE FIRST POSITION ON A BLADE       00004050
C     ENCOUNTERED FOR THE FIRST TIME.                                   00004060
C                                                                       00004070
      DIMENSION U22R(400,2),U22I(400,2),V22R(400,2),V22I(400,2),C22R(40000004080
     1,2),C22I(400,2),X(400,2),Y(400,2)                                 00004090
      DIMENSION U33R(400,2),U33I(400,2),V33R(400,2),V33I(400,2),C33R(40000004100
     1,2),C33I(400,2)                                                   00004110
      COMMON/BLK1/NGRDFN,FSTRMN,REDFRQ,XSUBO,TNWDST,FMANGL,XLNGTH,DELTAS00004120
     1,DSTSTR,HDSTRL,TRNGLH,U22R,U22I,V22R,V22I,C22R,C22I,X,Y,S,DELTA,  00004130
     2ISWTCH,JSWTCH,JLINE,IHAVEP,AI,BI,K12R,K12I,K34R,K34I,K56R,K56I,   00004140
```

```
     3VRPANL,VIPANL,STGANG,NSTPTS,LCOUNT,STAG                           00004150
      CCMMON/BLK5/U33R,U33I,V33R,V33I,C33R,C33I                         00004160
      CCMMON/BLADE/NUM,NUMBLD                                           00004170
      REAL K12R,K12I,K34R,K34I,K56R,K56I                                00004180
      NUM=NUM+1                                                         00004190
      XN=FLOAT(NUM)                                                     00004200
      I=IHAVEP+1                                                        00004210
      K12R=U22R(IHAVEP,JSWTCH)+C22R(IHAVEP,JSWTCH)+AI*U22I(IHAVEP,JSWTCH00004220
     1)                                                                 00004230
      K12I=-AI*U22R(IHAVEP,JSWTCH)+U22I(IHAVEP,JSWTCH)+C22I(IHAVEP,JSWTC00004240
     1H)                                                                00004250
12    K56R=U22R(IHAVEP,ISWTCH)+V22R(IHAVEP,ISWTCH)+BI*(U22I(IHAVEP,ISWTC00004260
     1H)-C22I(IHAVEP,ISWTCH))                                           00004270
      K56I=U22I(IHAVEP,ISWTCH)+V22I(IHAVEP,ISWTCH)-BI*(U22R(IHAVEP,ISWTC00004280
     1H)-C22R(IHAVEP,ISWTCH))                                           00004290
C                                                                       00004300
C     IF THE INTERBLADE PHASE ANGLE IS ZERO, SIMPLIFY THE NORMAL        00004310
C     VELOCITY COMPUTATION.                                             00004320
C                                                                       00004330
      IF(DELTA.LT.1.0E-05)GO TO 1                                       00004340
      V22R(I,ISWTCH)=-(COS(DELTA*XN)+XSUBO*REDFRQ*SIN(DELTA*XN))/S      00004350
      V22I(I,ISWTCH)=-(SIN(DELTA*XN)-XSUBO*REDFRQ*COS(DELTA*XN))/S      00004360
      GC TO 2                                                           00004370
    1 V22R(I,ISWTCH)=-1.0/S                                             00004380
      V22I(I,ISWTCH)=XSUBO*REDFRQ/S                                     00004390
    2 K34R=V22R(I,ISWTCH)                                               00004400
      K34I=V22I(I,ISWTCH)                                               00004410
      G1=1.0-AI*BI                                                      00004420
      G2=2.0*BI                                                         00004430
      G3=G1*G1+G2*G2                                                    00004440
      G4=K56R-K34R-BI*K12I                                              00004450
      G5=K56I-K34I+BI*K12R                                              00004460
      U22R(I,ISWTCH)=(G4*G1+G5*G2)/G3                                   00004470
      U22I(I,ISWTCH)=(-G4*G2+G5*G1)/G3                                  00004480
      C22R(I,ISWTCH)=K12R-U22R(I,ISWTCH)+AI*U22I(I,ISWTCH)              00004490
      C22I(I,ISWTCH)=K12I-U22I(I,ISWTCH)-AI*U22R(I,ISWTCH)              00004500
      K34R=U22R(I,JSWTCH)-V22R(I,JSWTCH)+BI*(U22I(I,JSWTCH)-C22I(I,JSWTC00004510
     1H))                                                               00004520
      K34I=U22I(I,JSWTCH)-V22I(I,JSWTCH)-BI*(U22R(I,JSWTCH)-C22R(I,JSWTC00004530
     1H))                                                               00004540
      V33R(I,ISWTCH)=V22R(I,ISWTCH)                                     00004550
      V33I(I,ISWTCH)=V22I(I,ISWTCH)                                     00004560
      K56R=V33R(I,ISWTCH)                                               00004570
      K56I=V33I(I,ISWTCH)                                               00004580
      G4=K56R+K34R-BI*K12I                                              00004590
      G5=K56I+K34I+BI*K12R                                              00004600
      U33R(I,ISWTCH)=(G4*G1+G5*G2)/G3                                   00004610
      U33I(I,ISWTCH)=(-G4*G2+G5*G1)/G3                                  00004620
```

```
      C33R(I,ISWTCH)=K12R-U33R(I,ISWTCH)+AI*U33I(I,ISWTCH)                00004630
      C33I(I,ISWTCH)=K12I-U33I(I,ISWTCH)-AI*U33R(I,ISWTCH)                00004640
      NUM=NUM+1                                                           00004650
      WRITE(6,100)NUM                                                     00004660
      NUM=NUM-1                                                           00004670
  100 FORMAT('0',10X,'BLADE #',I2,//)                                     00004680
  147 WRITE(6,145)X(I    ,ISWTCH),U22R(I    ,ISWTCH),U22I(I    ,ISWTCH),00004690
     1V22R(I    ,ISWTCH),V22I(I    ,ISWTCH),C22R(I    ,ISWTCH),C22I(I   00004700
     2 ,ISWTCH)                                                           00004710
      WRITE(6,145)X(I    ,ISWTCH),U33R(I    ,ISWTCH),U33I(I    ,ISWTCH),00004720
     1V33R(I    ,ISWTCH),V33I(I    ,ISWTCH),C33R(I    ,ISWTCH),C33I(I   00004730
     2 ,ISWTCH)                                                           00004740
  145 FORMAT(2H N,7E16.7,/)                                               00004750
      RETURN                                                              00004760
      END                                                                 00004770


      SUBROUTINE WAKE                                                     00004780
C                                                                         00004790
C     WAKE COMPUTES U,V, AND C ABOVE AND BELOW THE SLIP PLANE AFT OF A    00004800
C     BLADE.                                                              00004810
C                                                                         00004820
      DIMENSION A(8,8),B(8)                                               00004830
      DIMENSION U22R(400,2),U22I(400,2),V22R(400,2),V22I(400,2),C22R(40000004840
     1,2),C22I(400,2),X(400,2),Y(400,2)                                   00004850
      DIMENSION U33R(400,2),U33I(400,2),V33R(400,2),V33I(400,2),C33R(40000004860
     1,2),C33I(400,2)                                                     00004870
      COMMON/BLK1/NGRDFN,FSTRMN,REDFRQ,XSUBO,TNWCST,FMANGL,XLNGTH,DELTAS00004880
     1,DSTSTR,HDSTRL,TRNGLH,U22R,U22I,V22R,V22I,C22R,C22I,X,Y,S,DELTA,    00004890
     2ISWTCH,JSWTCH,JLINE,IHAVEP,AI,BI,K12R,K12I,K34R,K34I,K56R,K56I,     00004900
     3VRPANL,VIPANL,STGANG,NSTPTS,LCOUNT,STAG                             00004910
      COMMON/BLK5/U33R,U33I,V33R,V33I,C33R,C33I                           00004920
      REAL K12R,K12I,K34R,K34I,K56R,K56I                                  00004930
      I=IHAVEP                                                            00004940
      II=IHAVEP+1                                                         00004950
      J=ISWTCH                                                            00004960
      JJ=JSWTCH                                                           00004970
C                                                                         00004980
C     COMPUTE RIGHT HAND SIDE OF EQUATIONS.                               00004990
C                                                                         00005000
      B(1)=U22R(I ,JJ)+C22R(I ,JJ)+AI*U22I(I ,JJ)                         00005010
      B(2)=U22I(I ,JJ)+C22I(I ,JJ)-AI*U22R(I ,JJ)                         00005020
      B(3)=U22R(I ,J )+V22R(I ,J )+BI*(U22I(I ,J )-C22I(I ,J ))           00005030
      B(4)=U22I(I ,J )+V22I(I ,J )-BI*(U22R(I ,J )-C22R(I ,J ))           00005040
      B(5)=U33R(I ,JJ)+C33R(I ,JJ)+AI*U33I(I ,JJ)                         00005050
      B(6)=U33I(I ,JJ)+C33I(I ,JJ)-AI*U33R(I ,JJ)                         00005060
      B(7)=U22R(II,JJ)-V22R(II,JJ)+BI*(U22I(II,JJ)-C22I(II,JJ))           00005070
      B(8)=U22I(II,JJ)-V22I(II,JJ)-BI*(U22R(II,JJ)-C22R(II,JJ))           00005080
```

```
C                                                                       00005090
C     SET UP MATRIX OF COEFFICIENTS.                                    00005100
C                                                                       00005110
      DO 1 K=1,8                                                        00005120
      DO 1 L=1,8                                                        00C05130
      A(K,L)=0.0                                                        00005140
    1 CONTINUE                                                          00005150
      A(1,1)=1.0                                                        00005160
      A(1,7)=1.0                                                        00005170
      A(2,2)=1.0                                                        00005180
      A(2,8)=1.0                                                        00C05190
      A(3,1)=1.0                                                        00005200
      A(3,5)=1.0                                                        00005210
      A(4,2)=1.0                                                        00005220
      A(4,6)=1.0                                                        00005230
      A(5,3)=1.0                                                        00C05240
      A(5,7)=1.0                                                        00005250
      A(6,4)=1.0                                                        00005260
      A(6,8)=1.0                                                        00005270
      A(7,3)=1.0                                                        00005280
      A(8,4)=1.0                                                        00005290
      A(1,2)=-AI                                                        00005300
      A(5,4)=-AI                                                        00005310
      A(6,3)=AI                                                         00005320
      A(2,1)=AI                                                         00005330
      A(3,2)=-BI                                                        00005340
      A(4,7)=-BI                                                        00C05350
      A(7,4)=-BI                                                        00005360
      A(8,7)=-BI                                                        00005370
      A(3,8)=BI                                                         00C05380
      A(4,1)=BI                                                         00005390
      A(7,8)=BI                                                         00005400
      A(8,3)=BI                                                         00C05410
      A(7,5)=-1.0                                                       00005420
      A(8,6)=-1.0                                                       00005430
      CALL SIMQ(A,B,8,KS)                                               00005440
      U22R(II,J )=B(1)                                                  00005450
      U22I(II,J )=B(2)                                                  00005460
      U33R(II,J )=B(3)                                                  00005470
      U33I(II,J )=B(4)                                                  00005480
      V22R(II,J )=B(5)                                                  00005490
      V22I(II,J )=B(6)                                                  00005500
      C22R(II,J )=B(7)                                                  00005510
      C22I(II,J )=B(8)                                                  00C05520
      V33R(II,J )=B(5)                                                  00005530
      V33I(II,J )=B(6)                                                  00005540
      C33R(II,J )=B(7)                                                  00005550
      C33I(II,J )=B(8)                                                  00005560
```

```
      RETURN
      END
      SUBROUTINE BOTTOM
C
C     BOTTOM COMPUTES U,V, AND C AT A LOWER SURFACE POINT.
C
      DIMENSION U22R(400,2),U22I(400,2),V22R(40C,2),V22I(400,2),C22R(400
     1,2),C22I(400,2),X(400,2),Y(400,2)
      DIMENSION U33R(400,2),U33I(400,2),V33R(40C,2),V33I(400,2),C33R(400
     1,2),C33I(400,2)
      CCMMON/BLK1/NGRDFN,FSTRMN,REDFRQ,XSUBO,TNWCST,FMANGL,XLNGTH,DELTAS
     1,DSTSTR,HDSTRL,TRNGLH,U22R,U22I,V22R,V22I,C22R,C22I,X,Y,S,DELTA,
     2ISWTCH,JSWTCH,JLINE,IHAVEP,AI,BI,K12R,K12I,K34R,K34I,K56R,K56I,
     3VRPANL,VIPANL,STGANG,NSTPTS,LCOUNT,STAG
      CCMMON/BLK5/U33R,U33I,V33R,V33I,C33R,C33I
      REAL K12R,K12I,K34R,K34I,K56R,K56I
      I=IHAVEP
      J=JSWTCH
      II=IHAVEP+1
      JJ=ISWTCH
      K12R=U33R(I ,J )+C33R(I ,J )+AI*U33I(I ,J )
      K12I=U33I(I ,J )+C33I(I ,J )-AI*U33R(I ,J )
      K34R=U22R(II,J )-V22R(II,J )+BI*(U22I(II,J )-C22I(II,J ))
      K34I=U22I(II,J )-V22I(II,J )-BI*(U22R(II,J )-C22R(II,J ))
      V33R(II,JJ)=V22R(II,JJ)
      V33I(II,JJ)=V22I(II,JJ)
      K56R=V33R(II,JJ)
      K56I=V33I(II,JJ)
      G1=1.0-AI*BI
      G2=2.0*BI
      G3=G1*G1+G2*G2
      G4=K56R+K34R-BI*K12I
      G5=K56I+K34I+BI*K12R
      U33R( II , JJ    )=(G4*G1+G5*G2)/G3
      U33I(  II , JJ    )=(-G4*G2+G5*G1)/G3
      C33R(II,JJ)=K12R-U33R(II,JJ)+AI*U33I(II,JJ)
      C33I(II,JJ)=K12I-U33I(II,JJ)-AI*U33R(II,JJ)
      WRITE(6,100)X(II,ISWTCH),U33R(II,ISWTCH),U33I(II,ISWTCH),V33R(II,I
     1SWTCH),V33I(II,ISWTCH),C33R(II,ISWTCH),C33I(II,ISWTCH)
  100 FCRMAT(2H B,7E16.7,/)
      RETURN
```

```
C                             THE SAUER-HEINZ PROCEDURE
C
      REAL MACH,MANGL
      DIMENSION X(200,2),R(200,2),U(200,2),V(200,2),CP(200)
      CCMMON/BLK1/X,R,U,V,ISWTCH,JSWTCH,IHAVEP,CP
      CCMMON/BLK3/BETA,PI,HO,XM,RO
      COMMON/BLK4/KOUNT,DELTAX,DELTAY
      CCMMON/BLK5/MANGL
      READ(5,100)MACH,HO,RO,MFREQ,NGFN
      WRITE(6,101)NGFN,MACH,MFREQ,HO,RO
C
C     SET UP GRID AND LOGIC
C
      PI=3.141593
      BETA=SQRT(MACH**2-1.0)
      MANGL=ARSIN(1.0/MACH)
      FINGRD=FLOAT(NGFN)
      NUMPTS=NGFN+1
      S=0.5/COS(MANGL)
      DELTAS=S/FINGRD
      DELTAX=DELTAS*COS(MANGL)
      DELTAY=DELTAS*SIN(MANGL)
      XM=FLOAT(MFREQ)
      ISWTCH=2
      JSWTCH=1
      IHAVEP=0
      I=IHAVEP+1
C
C     INITIALIZE FLOW QUANTITIES
C
      X(I,JSWTCH)=0.0
      R(I,JSWTCH)=RO
      U(I,JSWTCH)=0.0
      V(I,JSWTCH)=0.0
      KCUNT=1
    2 CALL COMPXR
C
C     IF POINT IS NOT ON THE INITIAL MACH LINE, GO TO 3.
C
    9 IF(IHAVEP.NE.0)GO TO 3
      U(1,ISWTCH)=0.0
      V(1,ISWTCH)=0.0
      IHAVEP=IHAVEP+1
      GO TO 2
C
C     IF POINT IS ON THE SURFACE, CALL SURFAC.
C
    3 IF(IHAVEP.EQ.KOUNT)GO TO 4
```

```
      CALL GENFPT                                                       00000500
      IHAVEP=IHAVEP+1                                                   00000510
      GO TO 2                                                           00000520
    4 CALL SURFAC                                                       00000530
C                                                                       00000540
C     IF THIS IS THE LAST SURFACE POINT, TERMINATE THE PROGRAM.         00000550
C                                                                       00000560
      IF(X(IHAVEP+1,ISWTCH).GE.1.0)GO TO 7                              00000570
C                                                                       00000580
C     AFTER MEETING SURFACE, SWITCH LOGIC VARIABLES TO GO BACK          00000590
C     TO THE INITIAL MACH LINE.                                         00000600
C                                                                       00000610
      IHAVEP=0                                                          00000620
      IF(ISWTCH.EQ.1)GO TO 5                                            00000630
      ISWTCH=1                                                          00000640
      JSWTCH=2                                                          00000650
      GO TO 6                                                           00000660
    5 ISWTCH=2                                                          00000670
      JSWTCH=1                                                          00000680
    6 KOUNT=KOUNT+1                                                     00000690
      GO TO 2                                                           00000700
    7 CONTINUE                                                          00000710
  100 FORMAT(3F10.5,2I10)                                               00000720
  101 FORMAT('1',10X,'SUPERSONIC FLOW OVER A CYLINDRICAL SHELL: THE SAUE00000730
     1R-HEINZ PROCEDURE         ',//,15X,'GRID FINENESS=',I5,/,15X,'MACH00000740
     2 NUMBER=',F11.6,/,15X,'AXIAL MODE # =',I3  ,/,15X,'AMPLITUDE=',F1000000750
     3.5,/,15X,'RADIUS TO LENGTH RATIO=',F10.5,/)                       00000760
  102 FORMAT(F10.5)                                                     00000770
  103 FORMAT('0',                            ///,T6,'IHAVEP',T20,'X',T3000000780
     1,'R',T45,'U',T60,'V',T75,'CP',//)                                 00000790
      STOP                                                              00000800
      END                                                               00000810


      SUBROUTINE GENFPT                                                 00000820
C                                                                       00000830
C     GENFPT COMPUTES THE FLOW QUANTITIES IN THE GENERAL FLOW FIELD.    00000840
C                                                                       00000850
      DIMENSION X(200,2),R(200,2),U(200,2),V(200,2),CP(200)             00000860
      COMMON/BLK1/X,R,U,V,ISWTCH,JSWTCH,IHAVEP,CP                       00000870
      COMMON/BLK3/BETA,PI,HO,XM,RO                                      00000880
      COMMON/BLK4/KOUNT,DELTAX,DELTAY                                   00000890
      I=IHAVEP+1                                                        00000900
      RETA=(R(IHAVEP,ISWTCH)+R(I,ISWTCH))/2.0                           00000910
      RZETA=(R(I,JSWTCH)+R(I,ISWTCH))/2.0                               00000920
      A=V(IHAVEP,ISWTCH)-V(I,JSWTCH)+BETA*(RETA*U(IHAVEP,ISWTCH)+RZETA*U00000930
     1(I,JSWTCH))                                                       00000940
      B=BETA*(RZETA+RETA)                                               00000950
```

```
      U(I,ISWTCH)=A/B                                                   00000960
      A=RZETA*V(IHAVEP,ISWTCH)+RETA*V(I,JSWTCH)-BETA*RZETA*RETA*(U(I,JSW00000970
     1TCH)-U(IHAVEP,ISWTCH))                                            00000980
      B=RZETA+RETA                                                      00000990
      V(I,ISWTCH)=A/B                                                   00001000
      RETURN                                                            00001010
      END                                                               00001020


      SUBROUTINE SURFAC                                                 00001030
C                                                                       00001040
C     SURFAC COMPUTES THE FLOW QUANTITIES AT THE SURFACE.               00001050
C                                                                       00001060
      DIMENSION X(200,2),R(200,2),U(200,2),V(200,2),CP(200)             00001070
      CCMMON/BLK1/X,R,U,V,ISWTCH,JSWTCH,IHAVEP,CP                       00001080
      CCMMON/BLK3/BETA,PI,HO,XM,RO                                      00001090
      I=IHAVEP+1                                                        00001100
      SLP=HO*PI*XM*COS(XM*PI*X(I,ISWTCH))                               00001110
C     IF(IHAVEP.EQ.1)GO TO 1                                            00001120
      RETA=(R(IHAVEP,ISWTCH)+R(I,ISWTCH))/2.0                           00001130
      A=BETA*RETA*U(IHAVEP,ISWTCH)+V(IHAVEP,ISWTCH)-R(I,ISWTCH)*SLP     00001140
      B=BETA*RETA+SLP*R(I,ISWTCH)                                       00001150
      U(I,ISWTCH)=A/B                                                   00001160
      GO TO 2                                                           00001170
C                                                                       00001180
C ********************************************************************  00001190
C *    IF VALUES ARE NOT BEING COMPUTED AT THE SURFACE BUT AT A MEAN *  00001200
C *    SURFACE POSITION, THE FOLLOWING CARDS DESCRIBE THE VALUE OF   *  00001210
C *    RETA AND U.                                                   *  00001220
    1 H=R(2,ISWTCH)-RO                                                  00001230
      RETA=RO+(H/4.0)*(3.0+(1.0/(BETA*SLP)))                            00001240
      A=-R(2,ISWTCH)*SLP                                                00001250
      B=BETA*RETA+SLP*R(2,ISWTCH)                                       00001260
      U(2,ISWTCH)=A/B                                                   00001270
C *    TO USE THIS PROCEDURE, DELETE THE "C" IN CCLUMN 1 OF THE CARC *  00001280
C *    WHICH FOLLOWS THE DESCRIPTION OF "SLP" ABCVE.                 *  00001290
C ********************************************************************  00001300
C                                                                       00001310
    2 V(I,ISWTCH)=SLP*(1.0+U(I,ISWTCH))*R(I,ISWTCH)                     00001320
      CP(IHAVEP)=-2.0*U(I,ISWTCH)-(V(I,ISWTCH)/R(I,ISWTCH))**2+(U(I,ISWT00001330
     1CH)*BETA)**2                                                      00001340
      WRITE(6,100)IHAVEP,X(I,ISWTCH),R(I,ISWTCH),U(I,ISWTCH),V(I,ISWTCH)00001350
     1,CP(IHAVEP)                                                       00001360
  100 FCRMAT('0',T8,I3,T17,F7.5,T27,F9.6,T41,E11.5,T57,E11.5,T73,E11.5//00001370
     1)                                                                 00001380
      RETURN                                                            00001390
      END                                                               00001400
```

```
      SUBROUTINE COMPXR
C
C     COMPXR COMPUTES THE X,R POSITION OF THE GRID POINT AND DETERMINES
C     THE VALUE OF X AND R AT THE INTERSECTION OF THE RIGHT- RUNNING
C     MACH LINE WITH THE SURFACE.
C
      REAL MANGL
      DIMENSION X(200,2),R(200,2),U(200,2),V(200,2),CP(200),DELX(200),DE
     1LR(200)
      COMMON/BLK1/X,R,U,V,ISWTCH,JSWTCH,IHAVEP,CP
      COMMON/BLK3/BETA,PI,HO,XM,RO
      COMMON/BLK4/KOUNT,DELTAX,DELTAY
      COMMON/BLK5/MANGL
C
C     CHECK WHETHER POINT IS ON THE INITIAL MACH LINE.
C
      IF(IHAVEP.EQ.0)GO TO 1
      I=IHAVEP+1
C
C     CHECK WHETHER THIS IS A SURFACE POINT.
C
      IF(IHAVEP.EQ.KOUNT)GO TO 2
      X(I,ISWTCH)=X(IHAVEP,ISWTCH)+DELX(IHAVEP)
      R(I,ISWTCH)=R(IHAVEP,ISWTCH)-DELR(IHAVEP)
      RETURN
    1 X(1,ISWTCH)=X(1,JSWTCH)+DELTAX
      R(1,ISWTCH)=R(1,JSWTCH)+DELTAY
      RETURN
C
C     COMPUTE THE POINT OF INTERSECTION.
C
    2 YO=2.0*KOUNT*DELTAY
      XMAX=X(IHAVEP,ISWTCH)
    3 F=-TAN(MANGL)*XMAX-HO*SIN(XM*PI*XMAX)+YO
      FP=-TAN(MANGL)-HO*XM*PI*COS(XM*PI*XMAX)
      FAC=F/FP
      XMAX=XMAX-FAC
      IF(ABS(FAC).GT.0.00001)GO TO 3
      X(I,ISWTCH)=XMAX
      R(I,ISWTCH)=HO*SIN(XM*PI*XMAX)+RO
      DELX(KOUNT)=XMAX-X(IHAVEP,ISWTCH)
      DELR(KOUNT)=R(IHAVEP,ISWTCH)-R(I,ISWTCH)
      RETURN
      END
```

```
C
C                METHOD OF SINGULARITIES FOR TWO-BLADE CASCADE
C                     WITH SUBSONIC LEADING EDGE LCCUS
C
      REAL M
      DIMENSION PNR(2),PNI(2)
      COMMON/BLK1/PHIR,PHII,PHIXR,PHIXI,RF
      READ(5,100)M,RF,GRDFN,Y,STGR
      WRITE(6,101)M,RF,GRDFN,Y,STGR
      WRITE(6,104)
      BETA=SQRT(M**2-1.0)
      D=0.00001
      N=0
      DU=1.0/GRDFN
      X=Y*BETA
      STGR=STGR*0.01745
    4 CCNTINUE
      DC 2 J=1,2
      FOR=0.0
      FOI=0.0
      FOS=0.0
      FOT=0.0
      U=0.0
      AA=BETA*Y
      IF(J.EQ.1)GO TO 3
      X=X-Y*TAN(STGR)
      ARGC=0.0
      ARGD=0.0
      IF(X.EQ.0.0) GO TO 12
      IF(X.LT.0.0001)GO TO 7
      AA=0.0
    3 CONTINUE
      FAC=X-AA
      XX=FAC/DU+1.0
      LIMIT=XX
      DXX=XX-LIMIT
      IF(DXX.GE.0.5)LIMIT=LIMIT+1
      ARGA=(X-U)*RF*M**2/BETA**2
      ARGB=(RF*M/BETA**2)*SQRT(ABS((X-U)**2-AA**2))
      ARGC=X*RF*M**2/BETA**2
      ARGD=RF*M*SQRT(ABS(X**2-AA**2))/BETA**2
      IF(LIMIT.LE.1)GO TO 12
    8 CALL BESJ(ARGB,N,BJ,D,IER)
      A1=SIN(ARGA)*BJ*RF
      B1=SIN(ARGA)*BJ*RF
      C1=COS(ARGA)*BJ*RF
      D1=COS(ARGA)*BJ*RF
    9 DC 1 I=2,LIMIT
```

```
      U=U+DU
      ARGA=(X-U)*RF*M**2/BETA**2
      ARGB=(RF*M/BETA**2)*SQRT(ABS((X-U)**2-AA**2))
   10 CALL BESJ(ARGB,N,BJ,D,IER)
      A2=RF*(-RF*U*COS(ARGA)+SIN(ARGA))*BJ
      B2=BJ*SIN(ARGA)*RF
      C2=RF*( RF*U*SIN(ARGA)+COS(ARGA))*BJ
      D2=BJ*COS(ARGA)*RF
      FOR=FOR+(A1+A2)*DU/2.0
      FOI=FOI+(B1+B2)*DU/2.0
      FOS=FOS+(C1+C2)*DU/2.0
      FOT=FOT+(D1+D2)*DU/2.0
      A1=A2
      B1=B2
      C1=C2
      D1=D2
    1 CONTINUE
   12 CONTINUE
      CALL BESJ(ARGD,N,BJ,D,IER)
      PNR(J)=FOR+FOI+COS(ARGC)*BJ
      PNI(J)=FOS+FOT-SIN(ARGC)*BJ
      PNR(J)=PNR(J)/BETA
      PNI(J)=PNI(J)/BETA
      IF(J.EQ.2)GO TO 6
      GO TO 2
    7 PNR(2)=0.0
      PNI(2)=0.0
      PHIR=0.0
      PHII=0.0
      PHIXR=0.0
      PHIXI=0.0
      VSR=0.0
      VSI=0.0
      X=X+Y*TAN(STGR)
      GO TO 11
    6 CALL POTEN(X,VSR,VSI,M,BETA,STGR,Y,GRDFN)
   11 PN3R=PHIXR-RF*PHII
      PN3I=PHIXI+RF*PHIR
      PRESR=PNR(1)+PNR(2)-PN3R
      PRESI=PNI(1)+PNI(2)-PN3I
      WRITE(6,103)X,PNR(1),PNI(1),PNR(2),PNI(2),PHIR,PHII,PHIXR,PHIXI
      WRITE(6,102)PN3R,PN3I,VSR,VSI,PRESR,PRESI
    2 CONTINUE
      X=X+0.1
      IF(X.LE.Y*(TAN(STGR)+BETA))GO TO 4
  100 FORMAT(5F10.5)
  101 FORMAT('1',10X,'MACH NUMBER=',F8.6,/,11X,'REDFRQ=',F7.2,/,11X,'GRI
     1D FINENESS=',F7.2,/,11X,'Y=',F3.1,/,11X,'ZN(ZETA)=ZETA',/,11X,'BET
```

```
     1A=',F5.2///)                                                      00000970
  103 FORMAT('0',T5,F4.2,T13,F10.5,T27,F10.5,T41,F10.5,T55,F10.5,T69,F1000000980
     1.5,T83,F10.5,T97,F10.5,T111,F10.5)                                00000990
  102 FORMAT(' ',      T13,F10.5,T27,F10.5,T41,F10.5,T55,F10.5,T69,F1000001000
     1.5,T83,F10.5,T97,F10.5,T111,F10.5//)                              00001010
  104 FORMAT(' ',T7,'X',T16,'PNR(1)',T30,'PNI(1)',T44,'PNR(2)',T58,'PNI(00001020
     12)',T72,'PHIR',T86,'PHII',T100,'PHIXR',T114,'PHIXI',/,T16,'PNR(3)'00001030
     2,T30,'PNI(3)',T44,'VSR',T58,'VSI',T72,'PNR',T86,'PNI',/)          00001040
      STOP                                                              00001050
      END                                                               00001060


      SUBROUTINE VSIGMA(Z,VSR,VSI,M,BETA,STGR,Y,GRDFN)                  00001070
      REAL M                                                            00001080
      COMMON/BLK1/PHIR,PHII,PHIXR,PHIXI,RF                              00001090
      D=0.00001                                                         00001100
      N=1                                                               00001110
      DU=1.0/GRDFN                                                      00001120
      X=Z+Y*TAN(STGR)                                                   00001130
    4 CONTINUE                                                          00001140
      FOR=0.0                                                           00001150
      FOI=0.0                                                           00001160
      U=0.0                                                             00001170
      ARGA=(X-U)*RF*M**2/BETA**2                                        00001180
      ARGB=RF*M*SQRT(ABS((X-U)**2-(BETA*Y)**2))/BETA**2                 00001190
      ARGC=Y*RF*M**2/BETA**2                                            00001200
      CALL BESJ(ARGB,N,BJ,D,IER)                                        00001210
      A1= COS(ARGA)*BJ/SQRT(ABS((X-U)**2-(BETA)**2))                    00001220
      B1=-SIN(ARGA)*BJ/SQRT(ABS((X-U)**2-(BETA)**2))                    00001230
      XX=(X-BETA*Y)/DU+1.0                                              00001240
      LIMIT=XX                                                          00001250
      DXX=XX-LIMIT                                                      00001260
      IF(DXX.GE.0.5)LIMIT=LIMIT+1                                       00001270
      DO 1 I=2,LIMIT                                                    00001280
      U=U+DU                                                            00001290
      ARGA=(X-U)*RF*M**2/BETA**2                                        00001300
      ARGB=RF*M*SQRT(ABS((X-U)**2-(BETA*Y)**2))/BETA**2                 00001310
      CALL BESJ(ARGB,N,BJ,D,IER)                                        00001320
      A2=(RF*U*SIN(ARGA)+COS(ARGA))*BJ/SQRT(ABS((X-U)**2-(BETA*Y)**2))  00001330
      B2=(RF*U*COS(ARGA)-SIN(ARGA))*BJ/SQRT(ABS((X-U)**2-(BETA*Y)**2))  00001340
      FCR=FOR+(A1+A2)*DU/2.0                                            00001350
      FCI=FOI+(B1+B2)*DU/2.0                                            00001360
      A1=A2                                                             00001370
      B1=B2                                                             00001380
    1 CONTINUE                                                          00001390
      FCR=FOR*RF*M*Y/BETA                                               00001400
      FCI=FOI*RF*M*Y/BETA                                               00001410
      S=-RF*(X-Y*BETA)*SIN(ARGC)-COS(ARGC)                              00001420
```

```
      T=-RF*(X-Y*BETA)*COS(ARGC)+SIN(ARGC)
      VSR=FOR+S
      VSI=FOI+T
      RETURN
      END


      SUBROUTINE POTEN(X,VSR,VSI,M,BETA,STGR,Y,GRDFN)
      REAL M
      COMMON/BLK1/PHIR,PHII,PHIXR,PHIXI,RF
      U=0.0
      DU=1.0/GRDFN
      Z=0.0
      ARGA=(X-U)*RF*M**2/BETA**2
      ARGB=(X-U)*RF*M/BETA**2
      A1=0.0
      A2=0.0
      B1=0.0
      B2=0.0
      B3=0.0
      B4=0.0
      XX=X/DU+1.0
      LIMIT=XX
      DXX=XX-LIMIT
      IF(DXX.GE.0.5)LIMIT=LIMIT+1
      N=0
      CALL BESJ(ARGB,N,BJ1,D,IER)
      N=1
      CALL BESJ(ARGB,N,BJ2,D,IER)
      CALL VSIGMA(Z,VSR,VSI,M,BETA,STGR,Y,GRDFN)
      IF(LIMIT.LE.1)GO TO 2
      PR1=(VSR*COS(ARGA)+VSI*SIN(ARGA))*BJ1
      PI1=(VSI*COS(ARGA)-VSR*SIN(ARGA))*BJ1
      PP1=(VSR*SIN(ARGA)-VSI*COS(ARGA))*BJ1
      PP2=(VSR*CCS(ARGA)+VSI*SIN(ARGA))*BJ2
      PP3=(VSR*COS(ARGA)+VSI*SIN(ARGA))*BJ1
      PP4=(-VSR*SIN(ARGA)+VSI*COS(ARGA))*BJ2
      DO 1 I=2,LIMIT
      U=U+DU
      Z=U
      ARGA=(X-U)*RF*M**2/BETA**2
      ARGB=(X-U)*RF*M/BETA**2
      N=0
      CALL BESJ(ARGB,N,BJ1,D,IER)
      N=1
      CALL BESJ(ARGB,N,BJ2,D,IER)
      CALL VSIGMA(Z,VSR,VSI,M,BETA,STGR,Y,GRDFN)
      PR2=(VSR*COS(ARGA)+VSI*SIN(ARGA))*BJ1
```

```
      PI2=(VSI*COS(ARGA)-VSR*SIN(ARGA))*BJ1
      PP5=(VSR*SIN(ARGA)-VSI*COS(ARGA))*BJ1
      PP6=(VSR*COS(ARGA)+VSI*SIN(ARGA))*BJ2
      PP7=(VSR*COS(ARGA)+VSI*SIN(ARGA))*BJ1
      PP8=(-VSR*SIN(ARGA)+VSI*COS(ARGA))*BJ2
      A1=A1+(PR1+PR2)*DU/2.0
      A2=A2+(PI1+PI2)*DU/2.0
      B1=B1+(PP1+PP5)*DU/2.0
      B2=B2+(PP2+PP6)*DU/2.0
      B3=B3+(PP3+PP7)*DU/2.0
      B4=B4+(PP4+PP8)*DU/2.0
      PR1=PR2
      PI1=PI2
      PP1=PP5
      PP2=PP6
      PP3=PP7
      PP4=PP8
    1 CONTINUE
    2 PHIR=A1*(-1.0/BETA)
      PHII=A2*(-1.0/BETA)
      PHIXR=(M*B1+B2)*RF*M/(BETA**1.5)-(1.0/BETA)*VSR
      PHIXI=(M*B3+B4)*RF*M/(BETA**1.5)-(1.0/BETA)*VSI
      X=X+Y*TAN(STGR)
      RETURN
      END
```

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STRACT

Supersonic flow past oscillating cylindrical shells and oscillating flat-plate cascades is analyzed distributions and generalized aerodynamic forces are computed for arbitrary radius-to-length ratios. Axial and circumferential wave numbers, Mach number, and reduced frequencies. Excellent agreement is obtained with a previous solution of the complete unsteady linearized potential equation by Dowell and Widnall using Laplace transform techniques. Also, pressure distributions are computed for flat-plate cascades with subsonic leading edge locus for arbitrary solidity, stagger angle, frequency, and interblade phase angle. For comparison, a two-blade solution is developed using the method of singularities. Good agreement is obtained for this special case.

KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Method of Characteristics						
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